ISSN 2075-9827 **e-ISSN 2313-0210**
Carpathian Math. Publ. 2016, **8** (2), 215–223
doi:10.15330/cmp.8.2.215-223

http://www.journals.pu.if.ua/index.php/cmp
Карпатські матем. публ. 2016, Т.8, №2, С.215–223

DE N.

THE VERTEX ZAGREB INDICES OF SOME GRAPH OPERATIONS

Recently, Tavakoli et al. [6] introduced a new version of Zagreb indices, named as vertex Zagreb indices. In this paper explicit expressions of different graphs operations of vertex Zagreb indices are presented and also as an application, explicit formulas for vertex Zagreb indices of some chemical graphs are obtained.

Key words and phrases: degree, topological index, Zagreb index, graph operations.

Calcutta Institute of Engineering and Management, 24/1A Chandi Ghosh Road, Kolkata-700040, West Bengal, India
E-mail: de.nilanjan@rediffmail.com

INTRODUCTION

In this paper, all the graphs are simple connected, having no directed or weighted edges. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. Let the number of vertices and edges of G will be denoted by n and m respectively. Also let the edge connecting the vertices u and v is denoted by uv. The degree of a vertex v, is the number of first neighbors of v and is denoted by $d_G(v)$. Let $N(u)$ denotes the first neighbor set of u; then $|N(u)| = d_G(u)$. As usual P_n and C_n denote a path and cycle graph of order n respectively. Let, $\sum$ denotes the class of all graphs, then a function $T : \sum \to \mathbf{R}^+$ is known as a topological index if for every graph H isomorphic to G, $T(G) = T(H)$. Thus a topological index transforming chemical information of a molecular graph by means of a numeric parameter which characterize its topology and is necessarily invariant under automorphism of graphs.

The first and second Zagreb indices of a graph were introduced in 1972 [1], denoted by $M_1(G)$ and $M_2(G)$ and are respectively defined as

$$M_1(G) = \sum_{v\in V(G)} d_G(v)^2 = \sum_{uv\in E(G)} [d_G(u) + d_G(v)] \text{ and } M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v).$$

These indices are among one of the most important vertex-degree based topological indices and have good application, so that get lots of attention from chemists and mathematicians (see [2–5, 7]).

There are various study of different versions of Zagreb indices. One of the modified versions of classical Zagreb indices, the vertex version of first and second Zagreb indices were introduced by Tavakoli et al. in [6] to calculate the eccentric connectivity index and Zagreb coindices of graphs under generalized hierarchical product and are defined as

УДК 519.1
2010 *Mathematics Subject Classification:* 05C07, 05C35, 05C90.

© De N., 2016

$$\bar{M}_1^*(G) = \sum_{\{u,v\} \subseteq V(G)} [d_G(u) + d_G(v)], \quad \bar{M}_2^*(G) = \sum_{\{u,v\} \subseteq V(G)} d_G(u) d_G(v).$$

In that paper, they also derived explicit expressions of first and second vertex Zagreb indices of generalized hierarchical product graphs. Till date, the study of these indices are largely limited and hence we have attracted in studying mathematical properties of these vertex version of Zagreb indices.

Graph operations played a very important role in chemical graph theory, as some chemically interesting graphs can be obtained by different graph operations on some general or particular graphs. In [7], Khalifeh et al. derived some exact formula for computing first and second Zagreb indices under some graph operations. In [8], Ashrafi et al. presented some explicit formulae of Zagreb coindices under some graph operations. In [9], Das et al. derived some upper bounds for multiplicative Zagreb indices for different graph operations. In [10] and [11], the present author obtained F-index and F-coindex of different graph operations. In [12] the present author found reformulated first Zagreb index under different graph operations. In [13], Azari and Iranmanesh presented explicit formulas for computing the eccentric-distance sum of different graph operations. There are several other results regarding various topological indices under different graph operations are available in the literature (for details see [14–23]). In this paper, we derive some exact expression of the first and second vertex Zagreb indices of different graph operations such as union, join, Cartesian product, composition and corona product of graphs.

1 Main results

In this section, we study the first and second vertex Zagreb indices under union, join, Cartesian product, composition and corona product of graphs. All these operations are binary, and the join and Cartesian product of graphs are commutative operations, whereas the composition and corona product operations are noncommutative. Let G_1 and G_2 be two simple connected graphs, so that their vertex sets and edge sets are represented as $V(G_i)$ and $E(G_i)$ respectively, for $i \in \{1, 2\}$. Also let, n_i and m_i denote the number of vertices and edges of G_i respectively, for $i \in \{1, 2\}$.

1.1 Union

Definition 1.1. *The union of two graphs G_1 and G_2 is the graph denoted by $G_1 \cup G_2$ with the vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2)$. In this case we assume that $V(G_1)$ and $V(G_2)$ are disjoint.*

The degree of a vertex v of $G_1 \cup G_2$ is equal to degree of that vertex in the component G_i, $i = 1, 2$, that contains it. In the following we calculate the first and second vertex Zagreb indices of $G_1 \cup G_2$.

Theorem 1. *Let G_1 and G_2 be two connected graphs, then*

$$\bar{M}_1^*(G_1 \cup G_2) = \bar{M}_1^*(G_1) + \bar{M}_1^*(G_2) + 2n_2 m_1 + 2n_1 m_2.$$

Proof. From definition, it is clear that, the vertex Zagreb index of $G_1 \cup G_2$ is equal to the sum of the vertex Zagreb index of the components G_i, in addition to that the contributions of the missing edges between the components, which makes the edge set of the complete bipartite graph K_{n_1,n_2}. Thus we have

$$
\begin{aligned}
\bar{M}_1^*(G_1 \cup G_2) &= \sum_{\{u,v\} \in V(G_1)} \left[d_{G_1}(u) + d_{G_1}(v)\right] + \sum_{\{u,v\} \in V(G_2)} \left[d_{G_2}(u) + d_{G_2}(v)\right] \\
&+ \sum_{u \in V(G_1)} \sum_{v \in V(G_2)} \left[d_{G_1}(u) + d_{G_2}(v)\right],
\end{aligned}
$$

which proves the desired result. □

Theorem 2. *Let G_1 and G_2 be two connected graphs, then*

$$\bar{M}_2^*(G_1 \cup G_2) = \bar{M}_2^*(G_1) + \bar{M}_2^*(G_2) + 4m_1m_2.$$

Proof. From definition, similar to last theorem, we have

$$
\begin{aligned}
\bar{M}_2^*(G_1 \cup G_2) &= \sum_{\{u,v\} \in V(G_1)} d_{G_1}(u)d_{G_1}(v) + \sum_{\{u,v\} \in V(G_2)} d_{G_2}(u)d_{G_2}(v) \\
&+ \sum_{u \in V(G_1)} \sum_{v \in V(G_2)} d_{G_1}(u)d_{G_2}(v),
\end{aligned}
$$

which proves the desired result. □

1.2 Join

Definition 1.2. *The join of two graphs G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ is the graph denoted by $G_1 + G_2$ with the vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$.*

Thus in the sum of two graphs all the vertices of one graph are connected with all the vertices of the other graph, keeping all the edges of both graphs. So, the degree of the vertices of $G_1 + G_2$ is given by

$$
d_{G_1+G_2}(v) = \begin{cases} d_{G_1}(v) + n_2, & v \in V(G_1) \\ d_{G_2}(v) + n_1, & v \in V(G_2). \end{cases}
$$

In the following Theorem the first vertex Zagreb index of $G_1 + G_2$ is calculated.

Theorem 3. *The first vertex Zagreb index of $G_1 + G_2$ is given by*

$$\bar{M}_1^*(G_1 + G_2) = \bar{M}_1^*(G_1) + \bar{M}_1^*(G_2) + 2n_1m_2 + 2n_2m_1 + 2n_1n_2(n_1 + n_2 - 1).$$

Proof. Using definition of first vertex Zagreb index, we have

$$\bar{M}_1^*(G_1+G_2) = \sum_{\{u,v\}\subseteq V(G_1+G_2)} \left[d_{G_1+G_2}(u)+d_{G_1+G_2}(v)\right]$$

$$= \sum_{\{u,v\}\in V(G_1)} \left[d_{G_1+G_2}(u)+d_{G_1+G_2}(v)\right] + \sum_{\{u,v\}\subseteq V(G_2)} \left[d_{G_1+G_2}(u)+d_{G_1+G_2}(v)\right]$$

$$+ \sum_{u\in V(G_1),v\in V(G_2)} \left[d_{G_1+G_2}(u)+d_{G_1+G_2}(v)\right]$$

$$= \sum_{\{u,v\}\subseteq V(G_1)} \left[d_{G_1}(u)+d_{G_1}(v)+2n_2\right] + \sum_{\{u,v\}\subseteq V(G_2)} \left[d_{G_2}(u)+d_{G_2}(v)+2n_1\right]$$

$$+ \sum_{u\in V(G_1),v\in V(G_2)} \left[d_{G_1}(u)+n_2+d_{G_2}(v)+n_1\right]$$

$$= \sum_{\{u,v\}\subseteq V(G_1)} \left[d_{G_1}(u)+d_{G_1}(v)\right] + 2n_2.\frac{n_1(n_1-1)}{2}$$

$$+ \sum_{\{u,v\}\subseteq V(G_2)} \left[d_{G_2}(u)+d_{G_2}(v)\right] + 2n_1.\frac{n_2(n_2-1)}{2} + n_1n_2(n_1+n_2) + 2n_1m_2 + 2n_2m_1,$$

which proves the desired result. □

In the following, next we calculate the second vertex Zagreb index of G_1+G_2.

Theorem 4. *The second vertex Zagreb index of G_1+G_2 is given by*

$$\bar{M}_2^*(G_1+G_2) = \bar{M}_2^*(G_1) + n_2\bar{M}_1^*(G_1) + \bar{M}_2^*(G_2) + n_1\bar{M}_1^*(G_2) + \frac{1}{2}n_1{n_2}^2(n_1-1)$$
$$+ \frac{1}{2}{n_1}^2n_2(n_2-1) + 4m_1m_2 + 2n_1n_2(m_1+m_2) + {n_1}^2{n_2}^2.$$

Proof. Using definition of first vertex Zagreb index, we have

$$\bar{M}_2^*(G_1+G_2) = \sum_{\{u,v\}\subseteq V(G_1+G_2)} d_{G_1+G_2}(u)d_{G_1+G_2}(v)$$

$$= \sum_{\{u,v\}\subseteq V(G_1)} d_{G_1+G_2}(u)d_{G_1+G_2}(v) + \sum_{\{u,v\}\subseteq V(G_2)} d_{G_1+G_2}(u)d_{G_1+G_2}(v)$$

$$+ \sum_{u\in V(G_1),v\in V(G_2)} d_{G_1+G_2}(u)d_{G_1+G_2}(v) = \sum_{\{u,v\}\subseteq V(G_1)} (d_{G_1}(u)+n_2)(d_{G_1}(v)+n_2)$$

$$+ \sum_{\{u,v\}\subseteq V(G_2)} (d_{G_2}(u)+n_1)(d_{G_2}(v)+n_1) + \sum_{u\in V(G_1),v\in V(G_2)} (d_{G_1}(u)+n_2)(d_{G_2}(v)+n_1)$$

$$= \sum_{\{u,v\}\subseteq V(G_1)} d_{G_1}(u)d_{G_1}(v) + n_2 \sum_{\{u,v\}\subseteq V(G_1)} \left[d_{G_1}(u)+d_{G_1}(v)\right]$$

$$+ {n_2}^2.\frac{n_1(n_1-1)}{2} + \sum_{\{u,v\}\subseteq V(G_2)} d_{G_2}(u)d_{G_2}(v) + n_1 \sum_{\{u,v\}\subseteq V(G_2)} \left[d_{G_2}(u)+d_{G_2}(v)\right]$$

$$+ {n_1}^2.\frac{n_2(n_2-1)}{2} + \sum_{u\in V(G_1),v\in V(G_2)} (d_{G_1}(u)d_{G_2}(v) + n_1d_{G_1}(u) + n_2d_{G_2}(v) + n_1n_2)$$

$$= \bar{M}_2^*(G_1) + n_2\bar{M}_1^*(G_1) + \frac{1}{2}n_1{n_2}^2(n_1-1) + \bar{M}_2^*(G_2) + n_1\bar{M}_1^*(G_2)$$

$$+ \frac{1}{2}{n_1}^2n_2(n_2-1) + 4m_1m_2 + 2n_1n_2m_2 + 2n_1n_2m_1 + {n_1}^2{n_2}^2,$$

from where the desired result follows. □

Example 1. *The complete bipartite graph $K_{p,q}$ can be defined as $K_{p,q} = \bar{K}_p + \bar{K}_q$. So its vertex Zagreb indices can be calculated from the previous theorem as*

(i) $\bar{M}_1^*(K_{p,q}) = 2pq(p+q-1)$,

(ii) $\bar{M}_2^*(K_{p,q}) = pq\left[2pq - \frac{1}{2}(p+q)\right]$.

The suspension of a graph G is defined as sum of G with a single vertex. So from the previous proposition the following corollary follows.

Corollary 1.1. *The first and second vertex Zagreb indices of suspension of a graph G is given by*

(i) $\bar{M}_1^*(G+K_1) = \bar{M}_1^*(G) + 2n^2 + 2m$,

(ii) $\bar{M}_2^*(G+K_1) = \bar{M}_1^*(G) + \bar{M}_2^*(G) + 2mn + \frac{1}{2}n(3n-1)$.

Example 2. *The star graph S_n with n vertices is the suspension of empty graph $\bar{K}_{n-1}$. So its first and second vertex Zagreb indices can be respectively calculated from the previous corollary as*

(i) $\bar{M}_1^*(S_n) = 2(n-1)^2$,

(ii) $\bar{M}_2^*(S_n) = \frac{3}{2}(n-1)^2 - \frac{1}{2}(n-1)$.

Example 3. *The wheel graph W_n on $(n+1)$ vertices is the suspension of C_n. So from the previous corollary its first and second vertex Zagreb indices are given by*

(i) $\bar{M}_1^*(C_n+K_1) = 4n^2$,

(ii) $\bar{M}_2^*(C_n+K_1) = \frac{15}{2}n^2 - \frac{9}{2}n$.

Example 4. *The fan graph F_n on (n+1) vertices is the suspension of P_n. So from the previous corollary its first and second vertex Zagreb indices are given by*

(i) $\bar{M}_1^*(P_n+K_1) = 2n(2n-1)$,

(ii) $\bar{M}_2^*(P_n+K_1) = 2n(2n-1)$.

1.3 The Cartesian product

Definition 1.3. *Let G_1 and G_2 be two connected graphs. The Cartesian product of G_1 and G_2 denoted by $G_1 \times G_2$, is the graph with vertex set $V(G_1) \times V(G_2)$ and any two vertices (u_p, v_r) and (u_q, v_s) are adjacent if and only if [$u_p = u_q \in V(G_1)$ and $v_r v_s \in E(G_2)$] or [$v_r = v_s \in V(G_2)$ and $u_p u_q \in E(G_1)$] and $r, s = 1, 2, ..., |V(G_2)|$.*

In the following Theorem we express the first and second vertex Zagreb indices of the Cartesian product of graphs.

Theorem 5. *Let G_1 and G_2 be two connected graphs, then*

(i) $\bar{M}_2^*(G_1 \times G_1) = 2n_1n_2(n_1m_2 + n_2m_1) - 2m_1n_2 - 2m_2n_1$,

(ii) $\bar{M}_2^*(G_1 \times G_2) = 2(n_1m_2 + n_2m_1)^2 - 4m_1m_2 - \frac{1}{2}n_2M_1(G_1) - \frac{1}{2}n_1M_1(G_2)$.

The proof of the above Theorem follows by applying Theorem 1 and 4 of [7] and Proposition 13 of [8] respectively and using the fact that $M_1(G) = \bar{M}_1^*(G) - \bar{M}_1(G)$ and $M_2(G) = \bar{M}_2^*(G) - \bar{M}_2(G)$.

Example 5. *The Ladder graph L_n , made by n square and $(2n+2)$ vertices is the cartesian product of P_2 and P_{n+1}. So the first and second vertex Zagreb indices of L_n are given by*

(i) $\bar{M}_1^*(L_n) = 2(6n^2 + 5n + 1)$,

(ii) $\bar{M}_2^*(L_n) = 3(6n^2 + n + 1)$.

Example 6. *We have C_4-nanotorus $TC_4(m,n) = C_n \times C_m$. So its first and second vertex Zagreb indices are given by*

(i) $\bar{M}_1^*(TC_4(m,n)) = 4mn(mn-1)$,

(ii) $\bar{M}_2^*(TC_4(m,n)) = 8mn(mn-1)$.

Example 7. *We have C_4-nanotube $TUC_4(m,n) = P_n \times P_m$. So from the last theorem, its first and second vertex Zagreb indices are given by*

(i) $\bar{M}_1^*(TUC_4(m,n)) = 2(2mn - n - m)(mn-1)$,

(ii) $\bar{M}_2^*(TUC_4(m,n)) = 2(2mn - n - m) - 4(n-1)(m-1) - (4mn - 3(n+m))$.

1.4 Composition

Definition 1.4. *The composition or lexicographic product of two graphs G_1 and G_2 is denoted by $G_1[G_2]$ and any two vertices (u_1, u_2) and $(v_1, v_2)$are adjacent if and only if $u_1v_1 \in E(G_1)$or $[u_1 = v_1$ and $u_2v_2 \in E(G_2)]$.*

The vertex set of $G_1[G_2]$ is $V(G_1) \times V(G_2)$ and the degree of a vertex (a,b) of $G_1[G_2]$ is given by $d_{G_1[G_2]}(a,b) = n_2 d_{G_1}(a) + d_{G_2}(b)$.

The proof of the next Theorem follows similarly from the expressions of Zagreb indices and Zagreb coindices of composition of graphs from Theorem 3 and 6 of [7] and Proposition 18 of [8] respectively.

Theorem 6. *Let G_1 and G_2 be two connected graphs, then the first and second vertex Zagreb indices of $G_1[G_2]$ is given by*

$$(i)\ \bar{M}_1^*(G_1[G_2]) = 2n_1n_2(n_1m_2 + n_2{}^2m_1) - 2m_1(n_1 + n_2{}^2),$$

$$(ii)\ \bar{M}_2^*(G_1[G_2]) = 2m_1n_2{}^2(2n_1m_2 + n_2{}^2m_1) + 2n_1{}^2m_2{}^2 - 4m_1m_2n_2 - \frac{1}{2}n_2{}^3M_1(G_1) - \frac{1}{2}n_1M_1(G_2).$$

Example 8. *The fence graph is defined as $P_n[P_2]$. So from the last theorem its first and second vertex Zagreb indices are given by*

(i) $\bar{M}_1^*(P_n[P_2]) = 18n^2 - 22n + 8$,

(ii) $\bar{M}_2^*(P_n[P_2]) = 50n^2 - 105n + 64$.

Example 9. *The closed fence graph is defined as $C_n[P_2]$ so that from the last theorem its first and second vertex Zagreb indices are given by*

(i) $\bar{M}^*{}_1(C_n[P_2]) = 18n^2 - 8n$,

(ii) $\bar{M}_2^*(C_n[P_2]) = 18n^2 + 7n$.

1.5 Corona Product

The corona product $G_1 \circ G_2$ of two graphs G_1 and G_2 is obtained by taking one copy of G_1 and n_1 copies of G_2 and by joining each vertex of the i-th copy of G_2 to the i-th vertex of G_1, where $1 \leq i \leq n_1$. Thus, the corona product of G_1 and G_2 has total $(n_1n_2 + n_1)$ number of vertices and $(m_1 + n_1m_2 + n_1n_2)$ number of edges. A variety of topological indices under the corona product of graphs have already been studied by researchers [24, 26]. The degree of a vertex v of $G_1 \circ G_2$ is given by

$$d_{G_1 \circ G_2}(v) = \begin{cases} d_{G_1}(v) + n_2, & v \in V(G_1) \\ d_{G_2}(v) + 1, & v \in V(G_{2,i}),\ i = 1, 2, \ldots, n_1, \end{cases}$$

where, $G_{2,i}$ is the i-th copy of the graph G_2. In the following theorem, the first and second vertex Zagreb indices of the corona product of two graphs are computed. The proof of the following theorem follows by manipulating the definition of corona product of graphs and hence we omit it.

Theorem 7. *The first and second vertex Zagreb indices of $G_1 \circ G_2$ is given by*

$$\begin{aligned}(i)\ \bar{M}_1^*(G_1 \circ G_2) &= \bar{M}_1^*(G_1) + n_1\bar{M}_1^*(G_2) + 2n_1n_2[(n_2 + m_2)(n_1 - 1) + m_1 + n_1 + n_2 - 1] \\ &\quad + 2m_2{n_1}^2, \\ (ii)\ \bar{M}_2^*(G_1 \circ G_2) &= \bar{M}_2^*(G_1) + n_1\bar{M}_2^*(G_2) + 2{n_1}^2(n_2 + m_2)^2 + 2n_1m_1(n_2 + m_2) - 2n_1{m_2}^2 \\ &\quad - 2(n_1m_2 + n_2m_1) - \frac{1}{2}n_1n_2(n_2 + 1).\end{aligned}$$

Let for a graph G, n and m are number of vertices and edges of G, respectively. If degree of any end vertex of an edge is one then it is call a thorn or pendent edge. The t-thorny graph G^t of a given graph G is obtained by joining t-number of thorns to each vertex of G. Different topological indices of thorn graphs have already been studied by researcher (see [14,25,27,28]). We know that, the t-thorny graph of G is defined as the corona product of G and complement of complete graph with t vertices $\bar{K}_t$. So, from the previous theorem we get the following corollary.

Corollary 1.2. *The first and second vertex Zagreb indices of the t-thorny graph are given by*

(i) $\bar{M}_1^*(G^t) = \bar{M}_1^*(G) + 2nt(nt + n + m - 1)$,

(ii) $\bar{M}_2^*(G^t) = \bar{M}_2^*(G) + 2n^2t^2 - \frac{1}{2}nt^2 + 2mt(2n - 1) - \frac{1}{2}nt$.

where, n and m are number of vertices and edges of G, respectively.

Example 10. *The first and second vertex Zagreb indices of t-thorny graph of C_n are given by*

(i) $\bar{M}_1^*({C_n}^t) = 2n(n - 1) + 2nt(nt + 2n - 1)$,

(ii) $\bar{M}_2^*({C_n}^t) = 2n(n - 1) + nt(6n - \frac{1}{2}t - \frac{5}{2})$.

Example 11. *The first and second vertex Zagreb indices of t-thorny graph of P_n are given by*

(i) $\bar{M}_1^*({P_n}^t) = 2(n - 1)^2 + 2nt(nt + 2n - 2)$,

(ii) $\bar{M}_2^*(P_n{}^t) = 2n^2 - 6n + 2n^2t^2 + 4n^2t - \frac{1}{2}nt^2 - \frac{13}{2}nt + 2t + 5.$

Example 12. *Let, n and m are the number of vertices and edges of G, respectively. One of the hydrogen suppressed molecular graph is the bottleneck graph (B) of a given graph G, which is defined as the corona product of K_2 and G. Using last theorem, the first and second vertex Zagreb indices of bottleneck graph of G are given by*

(i) $\bar{M}_1^*(B) = 2\bar{M}_1^*(G) + 8n^2 + 4nm + 8n + 8m + 2,$

(ii) $\bar{M}_2^*(B) = 2\bar{M}_2^*(G) + 7n^2 + 4m^2 + 16nm + 5n + 4m + 1.$

2 CONCLUSION

In this paper, we have studied the first and second vertex Zagreb indices of different graph operations. Also we apply our results to compute the vertex Zagreb indices for some special classes of graphs and nano-structures. For further study, vertex Zagreb indices of some other graph operations and for different composite graphs can be computed.

REFERENCES

[1] Gutman I., Trinajstić N. *Graph theory and molecular orbitals. Total φ-electron energy of alternant hydrocarbons*. Chemical Physics Letters 1972, **17** (4), 535–538. doi:10.1016/0009-2614(72)85099-1

[2] Xu K., Tang K., Liu H., Wang J. *The Zagreb indices of bipartite graphs with more edges*. J. Appl. Math. Inf. 2015, **33** (3-4), 365–377. doi:10.14317/jami.2015.365

[3] Das K.C., Xu K., Nam J. *On Zagreb indices of graphs*. Front. Math. China 2015, **10** (3), 567–582. doi:10.1007/s11464-015-0431-9

[4] Zhou B. *Upper bounds for the Zagreb indices and the spectral radius of series-parallel graphs*. Int. J. Quantum Chem. 2007, **107** (4), 875–878. doi:10.1002/qua.21223

[5] Zhou B., Gutman I. *Further properties of Zagreb indices*. MATCH Commun. Math. Comput. Chem. 2005, **54**, 233–239.

[6] Tavakoli M., Rahbarnia F., Ashrafi A.R. *Eccentric connectivity and Zagreb coindices of the generalized hierarchical product of graphs*. J. Discrete Math. 2014, **2014**, article id 292679. doi:10.1155/2014/292679

[7] Khalifeha M.H., Yousefi-Azaria H., Ashrafi A.R. *The first and second Zagreb indices of some graph operations*. Discrete Appl. Math. 2009, **157** (4), 804–811. doi:10.1016/j.dam.2008.06.015

[8] Ashrafi A.R., Doslić T., Hamzeh A. *The Zagreb coindices of graph operations*. Discrete Appl. Math. 2010, **158** (15), 1571–1578. doi:10.1016/j.dam.2010.05.017

[9] Das K.C., Yurttas A., Togan M., Cevik A.S., Cangul I.N. *The multiplicative Zagreb indices of graph operations*. J. Inequal. Appl. 2013, 2013:90. doi:10.1186/1029-242X-2013-90

[10] De N., Nayeem S.M.A., Pal A. *F-index of some graph operations*. Discrete Math. Algorithm. Appl. 2016, **8** (2), article id 1650025. doi:10.1142/S1793830916500257

[11] De N., Nayeem S.M.A., Pal A. *The F-coindex of some graph operations*. SpringerPlus 2016, **5** (221). doi:10.1186/s40064-016-1864-7

[12] De N., Nayeem S.M.A. Pal A. *Reformulated Zagreb index of some graph operations*. Mathematics 2015, **3** (4), 945–960. doi:10.3390/math3040945

[13] Azari M., Iranmanesh A. *Computing the eccentric-distance sum for graph operations*. Discrete Appl. Math. 2013, **161** (18), 2827–2840. doi:10.1016/j.dam.2013.06.003

[14] De N., Pal A., Nayeem S.M.A. *Modified eccentric connectivity of generalized thorn graphs*. Int. J. Comput. Math. 2014, **2014**, article id 436140. doi:10.1155/2014/436140

[15] Azari M. *Sharp lower bounds on the Narumi-Katayama index of graph operations*. Appl. Math. Comput. 2014, **239**, 409–421. doi:10.1016/j.amc.2014.04.088

[16] Azari M., Iranmanesh A. *Some inequalities for the multiplicative sum Zagreb index of graph operations*. J. Math. Ineql. 2015, **9** (3), 727–738. doi:10.7153/jmi-09-60

[17] De N., Pal A., Nayeem S.M.A. *The irregularity of some composite graphs*. Int. J. Appl. Comput. Math. 2016, **2** (3), 411–420. doi:10.1007/s40819-015-0069-z

[18] De N., Pal A., Nayeem S.M.A. *On some bounds and exact formulae for connective eccentric indices of graphs under some graph operations*. Int. J. Comb. 2014, **2014**, article id 579257. doi:10.1155/2014/579257

[19] Khalifeh M.H., Yousefi-Azari H., Ashrafi A.R. *The hyper-Wiener index of graph operations*. Comput. Math. Appl. 2008, **56** (5), 1402–1407. doi:10.1016/j.camwa.2008.03.003

[20] Tavakoli M., Rahbarnia F., Ashrafi A.R. *Some new results on irregularity of graphs*. J. Appl. Math. Inf. 2014, **32**, 675–685. doi:10.14317/jami.2014.675

[21] Veylaki M., Nikmehr M.J., Tavallaee H.A. *The third and hyper-Zagreb coindices of some graph operations*. J. Appl. Math. Comput. 2016, **50** (1), 315–325. doi:10.1007/s12190-015-0872-z

[22] Eskender B., Vumar E. *Eccentric connectivity index and eccentric distance sum of some graph operations*. Trans. Comb. 2013, **2** (1), 103–111.

[23] De N., Nayeem S.M.A., Pal A. *Total eccentricity index of the generalized hierarchical product of graphs*. Int. J. Appl. Comput. Math. 2015, **1** (3), 503–511. doi:10.1007/s40819-014-0016-4

[24] Pattabiraman K., Kandan P. *Weighted PI index of corona product of graphs*. Discrete Math. Algorithm. Appl. 2014, **6**, article id 1450055. doi:10.1142/S1793830914500554

[25] Alizadeh Y., Iranmanesh A., Doslić T., Azari M. *The edge Wiener index of suspensions, bottlenecks, and thorny graphs*. Glas. Mat. 2014, **49** (1), 1–12.

[26] Yarahmadi Z., Ashrafi A.R. *The Szeged, Vertex PI, first and second Zagreb indices of corona product of graphs*. Filomat 2012, **26** (3), 467–472.

[27] De N. *On eccentric connectivity index and polynomial of thorn graph*. Appl. Math. 2012, **3** (8), 931–934. doi:10.4236/am.2012.38139

[28] De N. *Augmented eccentric connectivity index of some thorn graphs*. Intern. J. Appl. Math. Res. 2012, **1** (4), 671–680. doi:10.14419/ijamr.v1i4.326

Received 24.08.2016

Revised 25.09.2016

Де Н. *Індекси Загреба вершин для деяких операцій з графами* // Карпатські матем. публ. — 2016. — Т.8, №2. — С. 215–223.

Нещодавно Таваколі М. ввів новий клас індексів Загреба, які називаються індекси Загреба вершин. У цій статті подані явні вирази для різних операцій з графами та отримані формули для обчислення індексів Загреба вершин для деяких хімічних графів.

Ключові слова і фрази: степінь, топологічний індекс, індекс Загреба, операції з графами.