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A Kenmotsu metric as a conformal η-Einstein soliton

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The object of the present paper is to study some properties of Kenmotsu manifold whose metric is conformal η-Einstein soliton. We have studied certain properties of Kenmotsu manifold admitting conformal η-Einstein soliton. We have also constructed a 3-dimensional Kenmotsu manifold satisfying conformal η-Einstein soliton.

Key words and phrases: Einstein soliton, η-Einstein soliton, conformal η-Einstein soliton, η-Einstein manifold, Kenmotsu manifold.

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Introduction

The notion of Einstein soliton was introduced by G. Catino and L. Mazzieri [3] in 2016, which generates self-similar solutions to Einstein flow

$$\frac{\partial g}{\partial t} = -2\left(S - \frac{r}{2}g\right),$$

where S is Ricci tensor, g is Riemannian metric and r is the scalar curvature.

The equation of the η-Einstein soliton [2] is given by,

$$\pounds_\xi g + 2S + (2\lambda - r)g + 2\mu\eta \otimes \eta = 0,$$

where $\pounds_\xi$ is the Lie derivative along the vector field ξ, S is the Ricci tensor, r is the scalar curvature of the Riemannian metric g, and λ and μ are real constants. For $\mu = 0$, the data (g, ξ, λ) is called Einstein soliton.

In 2018, M.D. Siddiqi [6] introduced the notion of conformal η-Ricci soliton [8] as

$$\pounds_\xi g + 2S + \left[2\lambda - \left(p + \frac{2}{n}\right)\right] g + 2\mu\eta \otimes \eta = 0,$$

where $\pounds_\xi$ is the Lie derivative along the vector field ξ, S is the Ricci tensor, λ, μ are constants, p is a scalar non-dynamical field (time dependent scalar field) and n is the dimension of manifold. For $\mu = 0$, conformal η-Ricci soliton becomes conformal Ricci soliton [7].

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In [9], S. Roy, S. Dey and A. Bhattacharyya have defined conformal Einstein soliton, which can be written as

$$\pounds_V g + 2S + \left[2\lambda - r + \left(p + \frac{2}{n}\right)\right] g = 0, \tag{1}$$

where $\pounds_V$ is the Lie derivative along the vector field V, S is the Ricci tensor, r is the scalar curvature of the Riemannian metric g, λ is real constant, p is a scalar non-dynamical field (time dependent scalar field) and n is the dimension of manifold.

So we introduce the notion of conformal η-Einstein soliton as follows.

Definition 1. *A Riemannian manifold (M, g) of dimension n is said to admit conformal η-Einstein soliton if*

$$\pounds_\xi g + 2S + \left[2\lambda - r + \left(p + \frac{2}{n}\right)\right] g + 2\mu\eta \otimes \eta = 0, \tag{2}$$

where $\pounds_\xi$ is the Lie derivative along the vector field ξ, λ, μ are real contants and S, r, p, n are same as defined in (1).

In the present paper, we study conformal η-Einstein soliton on Kenmotsu manifold. The paper is organized as follows.

After introduction, section 2 is devoted for preliminaries on (2n+1)-dimensional Kenmotsu manifold. In section 3, we have studied conformal η-Einstein soliton on Kenmotsu manifold. Here we proved that if a (2n+1)-dimensional Kenmotsu manifold admits conformal η-Einstein soliton then the manifold becomes η-Einstein. We have also characterized the nature of the manifold if the manifold is Ricci symmetric and the Ricci tensor is η-recurrent. Also we have discussed the condition, when the manifold has cyclic Ricci tensor. Then we have obtained the conditons in a (2n+1)-dimensional Kenmotsu manifold admitting conformal η-Einstein soliton, when a vector field V is pointwise co-linear with ξ and a (0,2)-tensor field h is parallel with respect to the Levi-Civita connection associated to g. We have also examined the nature of a Ricci-recurrent Kenmotsu manifold admitting conformal η-Einstein soliton.

In last section, we have given an example of a 3-dimensional Kenmotsu manifold satisfying conformal η-Einstein soliton.

1 Preliminaries

Let M be a (2n+1)-dimensional connected almost contact metric manifold with an almost contact metric structure (ϕ, ξ, η, g), where ϕ is a $(1, 1)$-tensor field, ξ is a vector field, η is a 1-form and g is the compatible Riemannian metric such that

$$\phi^2(X) = -X + \eta(X)\xi, \;\; \eta(\xi) = 1, \;\; \eta \circ \phi = 0, \;\; \phi\xi = 0, \tag{3}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{4}$$

$$g(X, \phi Y) = -g(\phi X, Y),$$

$$g(X, \xi) = \eta(X), \tag{5}$$

for all vector fields $X, Y \in \chi(M)$.

The fundamental 2-form Φ on an almost contact metric manifold M^{2n+1} is defined by $\Phi(X, Y) = g(X, \phi Y)$ for any vector fields X and Y on M^{2n+1}. In an almost contact metric manifold, we have $\eta \wedge \Phi^n \neq 0$.

When $\Phi = d\eta$, an almost contact metric manifold becomes contact metric manifold.

An almost contact metric manifold satisfying $d\eta = 0$ and $d\Phi = 2\eta \wedge \Phi$ is said to be an almost Kenmotsu manifold [4].

An almost contact metric manifold is said to be a Kenmotsu manifold [5] if

$$(\nabla_X \phi)Y = -g(X, \phi Y)\xi - \eta(Y)\phi X,$$

$$\nabla_X \xi = X - \eta(X)\xi, \tag{6}$$

where ∇ denotes the Riemannian connection of g.

In a Kenmotsu manifold the following relations hold [1]:

$$\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X),$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{7}$$

$$R(X,\xi)Y = g(X,Y)\xi - \eta(Y)X,$$

where R is the Riemannian curvature tensor,

$$S(X,\xi) = -2n\eta(X), \tag{8}$$

$$S(\phi X, \phi Y) = S(X,Y) + 2n\eta(X)\eta(Y),$$

$$(\nabla_X \eta)Y = g(X,Y) - \eta(X)\eta(Y), \tag{9}$$

for all vector fields $X, Y, Z \in \chi(M)$.

Now we know,

$$(\pounds_\xi g)(X,Y) = g(\nabla_X \xi, Y) + g(X, \nabla_Y \xi), \tag{10}$$

for all vector fields $X, Y, \in \chi(M)$. Then using (6) and (10), we get,

$$(\pounds_\xi g)(X,Y) = 2[g(X,Y) - \eta(X)\eta(Y)]. \tag{11}$$

2 Conformal η-Einstein soliton on Kenmotsu manifold

Let M be a (2n+1)-dimensional Kenmotsu manifold. Consider the conformal η-Einstein soliton (2) on M as

$$(\pounds_\xi g)(X,Y) + 2S(X,Y) + \left[2\lambda - r + \left(p + \frac{2}{2n+1}\right)\right] g(X,Y) + 2\mu\eta(X)\eta(Y) = 0$$

for all vector fields $X, Y, \in \chi(M)$.

Theorem 1. *If the metric of a (2n+1)-dimensional Kenmotsu manifold is a conformal η-Einstein soliton, then the manifold becomes η-Einstein and the scalar curvature is*

$$\left(p + \frac{2}{2n+1}\right) - 4n + 2\lambda + 2\mu.$$

Proof. First, using (11), the above equation becomes,

$$S(X,Y) = -\left[\lambda - \frac{r}{2} + \frac{(p + \frac{2}{2n+1})}{2} + 1\right] g(X,Y) - (\mu - 1)\eta(X)\eta(Y). \tag{12}$$

Taking $Y = \xi$ in the above equation and using (8), we get,

$$r = \left(p + \frac{2}{2n+1}\right) - 4n + 2\lambda + 2\mu, \tag{13}$$

since $\eta(X) \neq 0$, for all $X \in \chi(M)$.

Also from (12), it follows that the manifold is η-Einstein and this completes the proof. □

Theorem 2. *If the metric of a (2n+1)-dimensional Ricci symmteric Kenmotsu manifold is a conformal η-Einstein soliton, then μ = 1 and the scalar curvature is $\left(p + \frac{2}{2n+1}\right) - 4n + 2\lambda + 2$.*

Proof. We know, that $(\nabla_X S)(Y,Z) = X(S(Y,Z)) - S(\nabla_X Y, Z) - S(Y, \nabla_X Z)$, for all vector fields X, Y, Z on M and ∇ is the Levi-Civita connection associated with g.

Now replacing the expression of S from (12), we obtain

$$(\nabla_X S)(Y,Z) = -(\mu - 1)[\eta(Z)(\nabla_X \eta)Y + \eta(Y)(\nabla_X \eta)Z] \tag{14}$$

for all vector fields X, Y, Z on M.

As the manifold M is Ricci symmetric, i.e $\nabla S = 0$.

Then from (14), we get

$$-(\mu - 1)[\eta(Z)(\nabla_X \eta)Y + \eta(Y)(\nabla_X \eta)Z] = 0$$

for all vector fields $X, Y, Z \in \chi(M)$.

Taking $Z = \xi$ in the above equation and using (9), (3), we obtain, $\mu = 1$.

Then from (13), we get

$$r = \left(p + \frac{2}{2n+1}\right) - 4n + 2\lambda + 2.$$

Hence, we complete the proof. □

Theorem 3. *If the metric of a (2n+1)-dimensional Kenmotsu manifold is a conformal η-Einstein soliton and the Ricci tensor S is η-recurrent, then the scalar curvature is*

$$2\lambda + 2\mu + \left(p + \frac{2}{2n+1}\right).$$

Proof. If the Ricci tensor S is η-recurrent, then we have $\nabla S = \eta \otimes S$, which implies

$$(\nabla_X S)(Y,Z) = \eta(X)S(Y,Z)$$

for all vector fields X, Y, Z on M. Using (14), the above equation reduces to

$$-(\mu - 1)[\eta(Z)(\nabla_X \eta)Y + \eta(Y)(\nabla_X \eta)Z] = \eta(X)S(Y,Z).$$

Taking $Y = \xi$, $Z = \xi$ in the above equation and using (9), (12), we get

$$\left[\lambda + \mu - \frac{r}{2} + \frac{p + \frac{2}{2n+1}}{2}\right] \eta(X) = 0,$$

which implies

$$r = 2\lambda + 2\mu + \left(p + \frac{2}{2n+1}\right).$$

This completes the proof. □

Theorem 4. *Let the metric of a (2n+1)-dimensional Kenmotsu manifold M is a conformal η-Einstein soliton. Then M has cyclic Ricci tensor if* $\mu = 1$.

Proof. Similarly from (14), we get

$$(\nabla_Y S)(Z,X) = -(\mu-1)[\eta(X)(\nabla_Y \eta)Z + \eta(Z)(\nabla_X \eta)Y], \tag{15}$$

and

$$(\nabla_Z S)(X,Y) = -(\mu-1)[\eta(Y)(\nabla_Z \eta)X + \eta(X)(\nabla_Z \eta)Y] \tag{16}$$

for all vector fields X, Y, Z on M.

Then adding (14), (15), (16) and using (9), (4), we obtain

$$\begin{aligned}(\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) = &-2(\mu-1)[\eta(X)g(\phi Y,\phi Z) \\ &+ \eta(Y)g(\phi Z,\phi X) + \eta(Z)g(\phi X,\phi Y)].\end{aligned} \tag{17}$$

Now, as the manifold M has cyclic Ricci tensor, i.e

$$(\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) = 0,$$

then from (17), we have

$$(\mu-1)[\eta(X)g(\phi Y,\phi Z) + \eta(Y)g(\phi Z,\phi X) + \eta(Z)g(\phi X,\phi Y)] = 0.$$

Taking $X = \xi$ in the above equation and using (3), we get $\mu = 1$.

Again, if we take $\mu = 1$ in (17), we obtaion $(\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) = 0$, i.e the manifold M has cyclic Ricci tensor and this completes the proof. □

Corollary 1. *If a (2n+1)-dimensional Kenmotsu manifold M has a cyclic Ricci tensor and the metric is a conformal η-Einstein soliton, then the scalar curvature is* $(p + \frac{2}{2n+1}) - 4n + 2\lambda + 2$.

Proof. If $\mu = 1$, then from (13) we obtain $r = \left(p + \frac{2}{2n+1}\right) - 4n + 2\lambda + 2$. □

Theorem 5. *Let M be a (2n+1)-dimensional Kenmotsu manifold admitting a conformal η-Einstein soliton* (g,V)*, V being a vector field on M. If V is pointwise co-linear with ξ, a vector field on M, then V is a constant multiple of ξ, provided the scalar curvature is*

$$2\lambda + 2\mu + \left(p + \frac{2}{2n+1}\right) - 4n.$$

Proof. A conformal η-Einstein soliton is defined on a (2n+1)-dimensional Kenmotsu manifold M as

$$\pounds_V g + 2S + \left[2\lambda - r + \left(p + \frac{2}{2n+1}\right)\right] g + 2\mu\eta\otimes\eta = 0, \tag{18}$$

where $\pounds_V$ is the Lie derivative along the vector field V, S is the Ricci tensor, r is the scalar curvature of the Riemannian metric g, λ, μ are real contants, p is a scalar non-dynamical field (time dependent scalar field).

Since, V is pointwise co-linear with ξ, let $V = b\xi$, where b is a function on M.

Then (18) becomes

$$(\pounds_{b\xi} g)(X,Y) + 2S(X,Y) + \left[2\lambda - r + \left(p + \frac{2}{2n+1}\right)\right] g(X,Y) + 2\mu\eta(X)\eta(Y) = 0$$

for all vector fields X, Y on M. Applying the property of Lie derivative and Levi-Civita connection, we have

$$bg(\nabla_X\xi, Y) + (Xb)\eta(Y) + bg(\nabla_Y\xi, X) + (Yb)\eta(X) + 2S(X,Y) \\ + \left[2\lambda - r + \left(p + \frac{2}{2n+1}\right)\right] g(X,Y) + 2\mu\eta(X)\eta(Y) = 0.$$

Now using (6), we get

$$2bg(X,Y) - 2b\eta(X)\eta(Y) + (Xb)\eta(Y) + (Yb)\eta(X) + 2S(X,Y) \\ + \left[2\lambda - r + \left(p + \frac{2}{2n+1}\right)\right] g(X,Y) + 2\mu\eta(X)\eta(Y) = 0.$$

Taking $Y = \xi$ in the above equation and using (3), (5), (8), we obtain

$$(Xb) + (\xi b)\eta(X) - 4n\eta(X) + \left[2\lambda - r + \left(p + \frac{2}{2n+1}\right)\right] \eta(X) + 2\mu\eta(X) = 0. \tag{19}$$

Then by putting $X = \xi$, the above equation reduces to

$$\xi b = 2n + \frac{r}{2} - \lambda - \mu - \frac{(p + \frac{2}{2n+1})}{2}. \tag{20}$$

Using (20), equation (19) becomes

$$(Xb) + \left[\lambda + \mu + \frac{(p + \frac{2}{2n+1})}{2} - 2n - \frac{r}{2}\right] \eta(X) = 0. \tag{21}$$

Applying exterior differentiation in (21), we have

$$\left[\lambda + \mu + \frac{(p + \frac{2}{2n+1})}{2} - 2n - \frac{r}{2}\right] d\eta = 0. \tag{22}$$

Now we know

$$d\eta(X,Y) = \frac{1}{2}\left[(\nabla_X\eta)Y - (\nabla_Y\eta)X\right]$$

for all vector fields X, Y on M. Using (9), the above equation becomes $d\eta = 0$. Hence the 1-form η is closed.

So from (22), either $r = 2\lambda + 2\mu + (p + \frac{2}{2n+1}) - 4n$ or $r \neq 2\lambda + 2\mu + (p + \frac{2}{2n+1}) - 4n$. If $r = 2\lambda + 2\mu + (p + \frac{2}{2n+1}) - 4n$, (21) reduces to $(Xb) = 0$. This implies that b is constant and this completes the proof. □

Theorem 6. *In a (2n+1)-dimensional Kenmotsu manifold assume that a symmetric (0,2)-tensor field $h = £_\xi g + 2S + 2\mu\eta \otimes \eta$ is parallel with respect to the Levi-Civita connection associated to g. Then (g, ξ) yields a conformal η-Einstein soliton.*

Proof. Note that h is a symmetric tensor field of (0,2)-type, which we suppose to be parallel with respect to the Levi-Civita connection ∇, i.e $\nabla h = 0$. Applying the Ricci commutation identity, we have

$$\nabla^2 h(X,Y;Z,W) - \nabla^2 h(X,Y;W,Z) = 0$$

for all vector fields X, Y, Z, W on M. From the above equation we obtain the relation

$$h(R(X,Y)Z,W) + h(Z,R(X,Y)W) = 0.$$

Replacing $Z = W = \xi$ in the above equation and using (7), we get

$$\eta(X)h(Y,\xi) - \eta(Y)h(X,\xi) = 0.$$

Replacing $X = \xi$ and using (3), the above equation reduces to

$$h(Y,\xi) = \eta(Y)h(\xi,\xi) \tag{23}$$

for all vector fields Y on M. Differentiating the above equation covariantly with respect to X, we get

$$\nabla_X(h(Y,\xi)) = \nabla_X(\eta(Y)h(\xi,\xi)).$$

Now expanding the above eqution by using (23), (6), (9) and the property $\nabla h = 0$, we obtain

$$h(X,Y) = h(\xi,\xi)g(X,Y) \tag{24}$$

for all vector fields X, Y on M.

Let us take

$$h = \pounds_\xi g + 2S + 2\mu\eta \otimes \eta. \tag{25}$$

Then from (11), (12), we get

$$h(\xi,\xi) = -2\lambda - \left(p + \frac{2}{2n+1}\right) + r.$$

Then using (25), equation (24) becomes

$$(\pounds_\xi g)(X,Y) + 2S(X,Y) + \left[2\lambda - r + \left(p + \frac{2}{2n+1}\right)\right] g(X,Y) + 2\mu\eta(X)\eta(Y) = 0,$$

which is the conformal η-Einstein soliton. Hence, we complete the proof. □

Definition 2. *A Kenmotsu manifold is said to be Ricci-recurrent manifold if there exists a non-zero 1-form A such that*

$$(\nabla_W S)(Y,Z) = A(W)S(Y,Z) \tag{26}$$

for any vector fields W, Y, Z on M.

Theorem 7. *If the metric of a (2n+1)-dimensional Ricci-recurrent Kenmotsu manifold is a conformal η-Einstein soliton with the 1-form A, then the scalar curvature becomes*

$$2\lambda + 2\mu + \left(p + \frac{2}{2n+1}\right) + 4n(A(\xi) - 1).$$

Proof. Replacing Z by ξ in the equation (26) and using (8), we get

$$(\nabla_W S)(Y,\xi) = -2nA(W)\eta(Y),$$

which implies that

$$WS(Y,\xi) - S(\nabla_W Y,\xi) - S(Y,\nabla_W \xi) = -2nA(W)\eta(Y).$$

Using (8) and (6), the above equation becomes

$$2n(\nabla_W \eta)Y + 2n\eta(W)\eta(Y) + S(Y,W) = 2nA(W)\eta(Y).$$

Again using (9), the above equation reduces to

$$2ng(W,Y) + S(Y,W) = 2nA(W)\eta(Y).$$

Taking $W = \xi$ in the above equation and using (12), we obtain

$$r = 2\lambda + 2\mu + \left(p + \frac{2}{2n+1}\right) + 4n(A(\xi) - 1).$$

This completes the proof. □

Example 1. *Here, we consider the three-dimensional manifold* $M = \{(x,y,z) \in \mathbb{R}^3, (x,y,z) \neq (0,0,0)\}$, *where* (x,y,z) *are standard coordinates in* $\mathbb{R}^3$. *The vector fields*

$$e_1 = z\frac{\partial}{\partial x}, \quad e_2 = z\frac{\partial}{\partial y}, \quad e_3 = -z\frac{\partial}{\partial z}$$

are linearly independent at each point of M. *Let* g *be the Riemannian metric defined by*

$$g(e_1,e_2) = g(e_2,e_3) = g(e_3,e_1) = 0, \qquad g(e_1,e_1) = g(e_2,e_2) = g(e_3,e_3) = 1.$$

Let η *be the 1-form defined by* $\eta(Z) = g(Z,e_3)$ *for any vector field* Z *in* $\mathbb{R}^3$ *and* ϕ *be the (1,1)-tensor field defined by* $\phi e_1 = -e_2$, $\phi e_2 = e_1$, $\phi e_3 = 0$. *Then using the linearity of* ϕ *and* g, *we have*

$$\eta(e_3) = 1, \quad \phi^2 Z = -Z + \eta(Z)e_3, \quad g(\phi Z, \phi W) = g(Z,W) - \eta(Z)\eta(W)$$

for any $Z, W \in \chi(M)$. *Thus for* $e_3 = \xi$, (ϕ, ξ, η, g) *defines an almost contact metric structure on* M.

Let ∇ *be the Levi-Civita connection with respect to the Riemannian metric* g. *Then we have* $[e_1,e_2] = 0$, $[e_1,e_3] = e_1$, $[e_2,e_3] = e_2$. *The connection* ∇ *of the metric* g *is given by*

$$2g(\nabla_X Y, Z) = Xg(Y,Z) + Yg(Z,X) - Zg(X,Y) - g(X,[Y,Z]) - g(Y,[X,Z]) + g(Z,[X,Y]),$$

which is known as Koszul's formula.

Using Koszul's formula, we can easily calculate

$$\begin{aligned}
&\nabla_{e_1} e_1 = -e_3, && \nabla_{e_1} e_2 = 0, && \nabla_{e_1} e_3 = e_1,\\
&\nabla_{e_2} e_1 = 0, && \nabla_{e_2} e_2 = -e_3, && \nabla_{e_2} e_3 = e_2,\\
&\nabla_{e_3} e_1 = 0, && \nabla_{e_3} e_2 = 0, && \nabla_{e_3} e_3 = 0.
\end{aligned}$$

From the above it follows that the manifold satisfies $\nabla_X \xi = X - \eta(X)\xi$ *for* $\xi = e_3$. *Hence the manifold is a Kenmotsu manifold. So, here we have considered* $\mathbb{R}^3$ *as an almost contact manifold, which turns out to be a 3-dimensional Kenmotsu manifold.*

Also, the Riemannian curvature tensor R *is given by*

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z.$$

Hence,

$$R(e_1,e_2)e_2 = -e_1, \quad R(e_1,e_3)e_3 = -e_1, \quad R(e_2,e_1)e_1 = -e_2,$$
$$R(e_2,e_3)e_3 = -e_2, \quad R(e_3,e_1)e_1 = -e_3, \quad R(e_3,e_2)e_2 = -e_3,$$
$$R(e_1,e_2)e_3 = 0, \qquad R(e_2,e_3)e_1 = 0, \qquad R(e_3,e_1)e_2 = 0.$$

Then, the Ricci tensor S is given by $S(e_1,e_1) = -2$, $S(e_2,e_2) = -2$, $S(e_3,e_3) = -2$. From (12), *we have*

$$S(e_3,e_3) = -\left[\lambda + \mu - \frac{r}{2} + \frac{(p+\frac{2}{3})}{2}\right],$$

which implies that

$$r = 2\lambda + 2\mu - 4 + \left(p + \frac{2}{3}\right).$$

Hence λ and μ satisfies equation (13) *and so g defines a conformal η-Einstein soliton on the 3-dimensional Kenmotsu manifold M.*

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Рой С., Дей С., Бхаттачарія А. *Метрика Кенмоцу як конформний η-солітон Айнштайна* // Карпатські матем. публ. — 2021. — Т.13, №1. — C. 110–118.

Метою даної роботи є вивчення деяких властивостей многовида Кенмоцу, метрика якого є конформним η-солітоном Айнштайна. Ми дослідили певні властивості многовида Кенмоцу, що допускає конформний η-солітон Айнштайна. Також ми збудували тривимірний многовид Кенмоцу, що задовольняє конформний η-солітон Айнштайна.

Ключові слова і фрази: солітон Айнштайна, η-солітон Айнштайна, конформний η-солітон Айнштайна, η-многовид Айнштайна, многовид Кенмоцу.