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Banach-Steinhaus theorem for linear relations on asymmetric normed spaces

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We study the continuity of linear relations defined on asymmetric normed spaces with values in normed spaces. We give some geometric charactirization of these mappings. As an application, we prove the Banach-Steinhaus theorem in the framework of asymmetric normed spaces.

Key words and phrases: linear relation, multivalued linear operator, asymmetric norm.

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Introduction

The properties of asymmetric normed spaces started with some papers (see [9–12]). Concerning the continuity of linear operators between asymmetric normed spaces, in spite of the existing differences, some results from the symmetric case have their counterparts in the asymmetric one, a study that was initiated in [6]. Also, a first study of continuous multilinear operators on asymmetric normed spaces is given in [7].

In the present work, we extend the notion of continuity of linear operators between asymmetric normed spaces to the linear relations (also known as multivalued linear operators) defined on asymmetric normed spaces. As far as we know that is a first attempt in this regard. After this introduction, in section two we extend to linear relations the concept of continuity in asymmetric normed spaces. We give a characterization of these mappings in a way analogous to that used to characterize linear relations between normed spaces. In Section 3, we establish the Banach-Steinhaus theorem for linear relations in asymmetric normed spaces.

The notation used in the paper is in general standard. A function $q\colon X \to \mathbb{R}_+$ on a real linear space X is an asymmetric norm if for every $x, y \in X$ and $\alpha \in \mathbb{R}_+$ the following hold:

(1) $q(x) = q(-x) = 0$ if and only if $x = 0$,

(2) $q(\alpha x) = \alpha q(x)$,

(3) $q(x+y) \leq q(x) + q(y)$.

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We say that the pair (X, q) is an asymmetric normed space.

The asymmetric norm conjugate to q is the function $\overline{q}\colon X \to \mathbb{R}_+$ defined by $\overline{q}(x) = q(-x)$. As a consequence, the asymmetric norm q induces a norm q^s defined on X by the formula $q^s(x) = \max\{q(x), q(-x)\}$, this norm is referred to as the symmetrization of the asymmetric norm q.

Let X be a nonempty set. A function $\rho\colon X \times X \to \mathbb{R}_+$, that satisfies the following conditions:

(1) $\rho(x, y) = \rho(y, x) = 0$ if and only if $x = y$,

(2) $\rho(x, y) \leq \rho(x, z) + \rho(z, y)$ for all $x, y, z \in X$,

is called a quasi-metric on X.

If ρ is a quasi-metric on a set X, then the function $\overline{\rho}$ defined on $X \times X$ by $\overline{\rho}(x, y) = \rho(y, x)$ for all $x, y \in X$, is also a quasi-metric on X called the conjugate with ρ, and the function ρ^s defined on $X \times X$ by $\rho^s(x, y) = \max\{\rho(x, y), \overline{\rho}(x, y)\}$ for all $x, y \in X$ is a metric on X.

Each asymmetric norm q on a linear space X induces a quasi-metric ρ_q on X defined by $\rho_q(x, y) = q(y - x)$ for all $x, y \in X$.

The asymmetric norm q induces a T_0 topology τ_q on X that is generated by the asymmetric open balls $B_q(x, \varepsilon) = \{y \in X : q(y - x) < \varepsilon\}$, where $x \in X$ and $\varepsilon > 0$. Moreover the collection $\{B_q(x, \varepsilon) : x \in X \text{ and } \varepsilon > 0\}$ forms a fundamental system of neighborhoods for the topology τ_q. However, in general this topology is not Hausdorff.

A sequence $(x_n)_n$ in an asymmetric normed space (X, q) is convergent to $x \in X$ with respect to τ_q if and only if $\lim\limits_{n \to +\infty} q(x_n - x) = 0$.

Example 1. *As an important example, let u be the asymmetric norm on the usual real linear space $\mathbb{R}$ defined by*

$$u(x) := x^+ = \max\{x, 0\}. \tag{1}$$

In this case $\overline{u} = \max\{-x, 0\} = x^-$ and $u^s = \max\{-x, x\} = |x|$. Obviously $(\mathbb{R}, u)$ is an asymmetric normed space.

Let (X, q) and (Y, p) be asymmetric normed spaces. We denote by $LC(X, Y)$ the set of all continuous linear mappings from (X, q) to (Y, p) and by $LC^s(X, Y)$ the set of all continuous linear mappings from (X, q^s) to (Y, p^s). The set $LC(X, Y)$ is not necessarily a linear space but it is a cone (or normed semilinear space) with $LC(X, Y) \subset LC^s(X, Y)$ (see [6]).

The next result and its consequences can be found in [5] and [6] and will be used in the sequel.

Proposition 1. *A linear mapping T belongs to $LC(X, Y)$ if and only if there is a constant $M > 0$ such that $p(T(x)) \leq Mq(x)$ for all $x \in X$.*

Following [6, Theorem 1], we can consider the asymmetric norm on the cone $LC(X, Y)$ of all linear continuous mappings T from (X, q) into (Y, p) defined by the formula

$$\|T| := \sup\{p(T(x)) : q(x) \leq 1\},$$

and one can easily show that

$$\|T| = \inf\{M > 0 : p(T(x)) \leq Mq(x)\}.$$

For the general theory of asymmetric normed spaces we refer the reader to the monograph [1].

Regarding the normed linear relations, a linear relation (is also called a multivalued linear operator) T between two real linear spaces X and Y is a mapping defined on a subspace $D(T)$ of X, called the domain of T, which takes on values in the collection of non-empty subsets of Y such that

$$T(\alpha x_1 + \beta x_2) = \alpha T(x_1) + \beta T(x_2)$$

for all nonzero real numbers α, β and $x_1, x_2 \in D(T)$. If T maps the points of its domain to singletons, then T is said to be a single-valued or simply an operator. The class of all linear relations from X to Y is denoted by $LR(X,Y)$. In the case where $X = Y$, briefly by $LR(X)$ we denote $LR(X,X)$. A linear relation $T \in LR(X,Y)$ is uniquely determined by its graph $G(T)$, which is defined by

$$G(T) = \{(x,y) \in X \times Y : x \in D(T),\ y \in Tx\},$$

so that we can identify T with $G(T)$. The inverse of T is the linear relation T^{-1} defined by

$$G(T^{-1}) = \{(y,x) \in Y \times X : (x,y) \in G(T)\}.$$

If T^{-1} is single-valued, then T is called injective.

The image of $A \subset X$ is given by

$$T(A) = \bigcup \{Tx : x \in A \cap D(T)\}.$$

So the range of T is defined by $R(T) := T(X)$. In particular, the multivalued part of T is defined by

$$T(0) := \{y : (0,y) \in G(T)\}.$$

It is worth mentioning that $T(0)$ is a linear subspace of Y (see [2, Corollary I.2.4]). Proposition I.2.8 in [2] gives a formula that charactrize the elements $y \in Tx$ for all $x \in D(T)$, namely

$$Tx = y + T(0) \tag{2}$$

If $R(T) = Y$, then T is called surjective.

For a non-empty subsets $B \subset Y$, set

$$T^{-1}(B) = \{x \in D(T) : B \cap Tx \neq \varnothing\}.$$

In particular, for $y \in R(T)$,

$$T^{-1}y = \{x \in D(T) : y \in Tx\}.$$

Note that T is injective if and only if its null space

$$N(T) := T^{-1}(0) = \{x \in D(T) : (x,0) \in G(T)\}$$

is reduced to $\{0\}$.

By [2, Proposition I.3.1], if $B \subset Y$ with $B \neq \varnothing$ and $A \subset X$, then we have

$$T\left(T^{-1}(B)\right) = B \cap R(T) + T(0) \tag{3}$$

and

$$T^{-1}(T(A)) = A \cap D(T) + T^{-1}(0). \tag{4}$$

All the other relevant terminology and preliminaries that we will use are given in corresponding sections. For the theory of multivalued linear operators we refer to the book of R.W. Cross [2].

1 Continuous linear relations

Let $T\colon (X,q) \to (Y,p)$ be a relation between asymmetric normed spaces. For defining the boundedness of T, we need the quotient space $Y/\overline{T(0)}$, but in the framework of asymmetric normed spaces the closure $\overline{T(0)}$ may fail to be a linear subspace of Y (see [3, Example 2.1]). For this reason, we consider linear relations acting from an asymmetric normed space to a normed space only instead of those acting between two asymmetric normed spaces.

We give a characterization of continuous linear ralations defined on asymmetric normed spaces with values in a normed spaces in a way analogous to that used to characterize it between normed spaces. The definition of a continuous linear relation is similar to that, given for the case of topological spaces in [2, Definition II.3.1]. We write the definition for the aim of completeness.

Definition 1. *A linear ralation $T \in RL(X,Y)$ between asymmetric normed space (X,q) and normed space $(Y,\|\cdot\|)$ is said to be continuous if for each neighbourhood V in $R(T)$ the inverse image $T^{-1}(V)$ is a neighbourhood in $D(T)$. Also T is called open if whenever U is a neighbourhood in $D(S)$, the image $T(U)$ is a neighbourhood in $R(T)$.*

Let $T \in LR(X,Y)$. Consider the quotient space $Y/\overline{T(0)}$ equiped with the norm

$$\|\bar{y}\| = \inf_{k\in\overline{T(0)}} \|y+k\| .$$

It is easy to prove that if $Q : Y \longrightarrow Y/\overline{T(0)}$ is the quotient map, then QT is single-valued (see [2, Proposition II.1.2]). Define

$$\|Tx\| := \|QTx\| = \inf_{k\in\overline{T(0)}} \|y+k\|$$

for all $x \in D(T)$ and $y \in Tx$. Also we put $\|T|_q := \|QT|$. Consequently, $\|T|_q = \sup_{q(x)\le 1} \|QTx\|$.

If M is a subspace of X (or Y) and α is an asymmetric norm on X or a norm on Y we consider

$$B_M = \{x \in M : \alpha(x) \le 1\}, \quad U_M = \{x \in M : \alpha(x) < 1\} .$$

These notations open the door to characterize the continuity of a linear relation $T \in LR(X,Y)$ by using the number $\|T|_q$. For the proof we need the following geometric charactirization of $\|T|_q$.

Proposition 2. *Let $T \in RL(X,Y)$ be a relation between asymmetric normed space (X,q) and normed space $(Y,\|\cdot\|)$. The following statements are equivalent.*

(i) $\|T|_q < \infty$.

(ii) There is a number $\lambda > 0$ such that

$$T\left(B_{D(T)}\right) \subset \lambda B_{R(T)} + T(0). \tag{5}$$

Proof. *(i)* $\Longrightarrow$ *(ii)*. Take $y \in T(B_{D(T)})$, then there is $x \in B_{D(T)}$ such that $y \in Tx$. It follows from $\|T|_q < \infty$, that

$$\|Tx\| = \inf_{k\in\overline{T(0)}} \|y+k\| = \inf_{k\in T(0)} \|y+k\| < \infty.$$

Note that the second equality is an immediate consequence of the continuity of the norm. Fix $\varepsilon > 0$, choose $k \in T(0)$ such that $\|y+k\| < \|T|_q + \varepsilon$. This means that $y+k \in \lambda B_{R(T)}$ with $\lambda = \|T|_q + \varepsilon$, therefore $y \in \lambda B_{R(T)} + T(0)$.

(ii) $\Longrightarrow$ *(i)*. Suppose that (5) holds for a given $\lambda > 0$. Let $x \in B_{D(T)}$ and $y \in Tx$. By the hypothesis there exist $y_1 \in B_{R(T)}$ and $k_1 \in T(0)$ such that $y = \lambda y_1 + k_1$. Thus, by (2) we get

$$\|Tx\| = \|QTx\| = \|Qy\| = \inf_{k \in T(0)} \|y+k\| \leq \|y - k_1\| = \lambda \|y_1\| \leq \lambda.$$

Consequently, $\|T|_q = \sup_{q(x)\leq 1} \|Tx\| \leq \lambda < \infty$. □

Theorem 1. *Let (X,q) be an asymmetric normed space and $(Y, \|\cdot\|)$ be a normed space. Then the linear relation $T\colon X \to Y$ is continuous if and only if $\|T|_q < \infty$.*

Proof. For the "if" part, suppose that $\|T|_q < \infty$. Let V be an open ball in $R(T)$ with center y. Then for some $\alpha > 0$ we can write $V - \{y\} = \alpha U_{R(T)}$. By (5) there exists $\lambda > 0$ such that $T(U_{D(T)}) \subset \lambda U_{R(T)} + T(0)$. An application of equality (4) reveals that

$$U_{D(T)} + T^{-1}(0) \subset \lambda T^{-1}(U_{R(T)}) = \frac{\lambda}{\alpha} T^{-1}(V - \{y\}).$$

Thus

$$\frac{\alpha}{\lambda} U_{D(T)} + T^{-1}(y) \subset T^{-1}(V),$$

which means that $T^{-1}(V)$ is a neighbourhood in $D(T)$. Therefore T is continuous.

To prove the "only if" part, assume that T is continuous. By using the preceding proposition, it is enough to show inclusion (5) for some $\lambda > 0$. There exists an open neighborhood O in $D(T)$ such that $O \subset T^{-1}U_{R(T)}$. Hence

$$O - O \subset T^{-1}\left(U_{R(T)}\right) - T^{-1}\left(U_{R(T)}\right).$$

On the other hand we have

$$T^{-1}\left(U_{R(T)}\right) - T^{-1}\left(U_{R(T)}\right) = T^{-1}\left(U_{R(T)} - U_{R(T)}\right) = 2T^{-1}\left(U_{R(T)}\right),$$

which means that $2T^{-1}\left(U_{R(T)}\right)$ is an open neighborhood of $\{0\}$ in $D(T)$. Consequently, there exists $\lambda > 0$ such that

$$\frac{4}{\lambda} U_{D(T)} \subset 2T^{-1}\left(U_{R(T)}\right).$$

It follows from $B_{D(T)} \subset 2U_{D(T)}$ and $U_{R(T)} \subset B_{R(T)}$ that

$$\frac{2}{\lambda} B_{D(T)} \subset 2T^{-1}\left(B_{R(T)}\right).$$

By (3) we get

$$\frac{2}{\lambda} T(B_{D(T)}) \subset 2T\left(T^{-1}\left(B_{R(T)}\right)\right) = 2B_{R(T)} + T(0),$$

and we obtain

$$T(B_{D(T)}) \subset \lambda B_{R(T)} + T(0).$$

□

By $CR(X,Y)$ we denote the set of all continuous linear relations between the asymmetric normed space (X,q) and the normed space $(Y,\|\cdot\|)$ and by $CR^s(X,Y)$ the normed linear space of all linear relations between the normed linear spaces (X,q^s) and $(Y,\|\cdot\|)$.

Proposition 3. *Let $T \in CR(X,Y)$. Then*

$$\|Tx\| \leq \|T|_q \, q(x) \tag{6}$$

for all $x \in X$. Moreover, $\|T|_q$ can be calculated also by the formula

$$\|T|_q = \inf\{M : \|Tx\| \leq Mq(x) \text{ for all } x \in X\}. \tag{7}$$

Proof. For every $x \in X$ such that $q(x) \neq 0$ from $\|T|_q = \sup_{q(x)\leq 1} \|Tx\|$ we get

$$\left\| T\frac{x}{q(x)} \right\| \leq \|T|_q \, ,$$

and we obtain inequality (6). If $q(x)=0$, the inequality is obvious. On the other hand, if λ is the right side member of equality (7), then it is clear that $\lambda \leq \|T|_q$. For the reverse inequality, if $M > 0$ satisfies $\|Tx\| \leq Mq(x)$ for all $x \in X$, it follows that $\|T|_q = \sup_{q(x)\leq 1} \|Tx\| \leq M$ and so $\|T|_q \leq \lambda$. □

For the proof of the following corollary, we use Theorem 1.

Corollary 1. *A linear relation $T\colon X \to Y$ is continuous from (X,q) to $(Y,\|\cdot\|)$ if and only if it is continuous from $(X,\overline{q})$ to $(Y,\|\cdot\|)$. Hence $CR(X,Y) \subset CR^s(X,Y)$.*

Proof. The required equivalence follows from

$$\sup_{\overline{q}(x)\leq 1} \|QTx\| = \sup_{q(x)\leq 1} \|QT(-x)\| = \sup_{q(x)\leq 1} \|QTx\| \, .$$

With this we have $\|T|_q = \|T|_{\overline{q}}$. Now let $T \in CR(X,Y)$. For all $x \in X$ such that

$$q^s(x) = \max\{q(x), \overline{q}(x)\} \leq 1,$$

we have $\|T\| = \sup_{q^s(x)\leq 1} \|T(x)\| \leq \|T|_q < \infty$ and the proof follows. □

Now we give an example of a continuous linear relation.

Example 2. *We can define a linear relation $T\colon (\mathbb{R},u) \to (\mathbb{R}^2,\|\cdot\|)$ by*

$$T(x) = \left\{(a,b) \in \mathbb{R}^2 : a+b = x\right\},$$

where u is the usual asymmetric norm on $\mathbb{R}$ defined by (1) and $\|\cdot\|$ is the norm on $\mathbb{R}^2$ defined by $\|(a,b)\| = |a| + |b|$. Let us show that T is continuous. Firstly, it is easy to see that $T(0)$ is a closed subspace of $\mathbb{R}^2$ and by simple calculation we can see that

$$\|T(x)\| = \inf_{(k_1,k_2)\in\overline{T(0)}} \|(a+k_1, b+k_2)\| = \inf_{k\in\mathbb{R}} \{|a+k| + |b-k|\} = |a+b|$$

for all $x \in \mathbb{R}$, $(a,b) \in T(x)$. It follows that $\|T|_u = \sup_{(a+b)^+\leq 1} |a+b| \leq 1$.

2 Banach-Steinhaus theorem

In this section, we present a Banach-Steinhaus theorem for continuous linear relations defined on asymmetric normed spaces. Recall that an asymmetric normed space (X, q) is said to be of the half second category if the condition $X = \cup_{n\geq 1} E_n$ implies $int_q\left(cl_{\overline{q}}(E_m)\right) \neq \varnothing$ for some $m \in \mathbb{N}$, where $int_q(A)$ is the interior of the set A in the topological space (X, τ_q) and $cl_{\overline{q}}(A)$ is the closure of A in the topological space $(X, \tau_{\overline{q}})$. Note that if q is a norm on X, the notion of space of the half second category coincides with the classical notion of a space of the second category (see [4] or [8]).

The next result, an asymmetric version of the Banach-Steinhaus theorem for linear operators, can be found in [4] and will be generalized to the setting of linear relations.

Theorem 2 ([4, Theorem 2.6]). *Let (X, p) and (Y, q) be two asymmetric normed spaces. Suppose that (X, p) is of the half second category. If $\mathcal{F}$ is a family of continuous linear operators such that $\sup\limits_{T\in\mathcal{F}} q(T(x)) < \infty$ for every $x \in X$, then*

$$\sup\{q(T(x)) : p(x) \leq 1\} < \infty.$$

For the proof of the main theorem we need the following lemma.

Lemma 1 ([4, Lemma 2.4]). *If (X, q) is an asymmetric normed space of the half second category and $\mathcal{F}$ is a family of real valued lower semicontinuous functions on the quasi metric space $(X, \bar{\rho_q})$ such that for each $x \in X$ there exists $b_x > 0$ such that $f(x) \leq b_x$ for all $f \in \mathcal{F}$, then there exist a nonempty open set U in (X, q) and $b > 0$ such that $f(x) \leq b$ for all $f \in \mathcal{F}$ and $x \in U$.*

Theorem 3. *Let (X, q) be an asymmetric normed space of the half second category, $(Y, \|\cdot\|)$ be a normed space and $\mathcal{F}$ be a family of continuous linear relations from (X, q) to $(Y, \|\cdot\|)$. Suppose that $\mathcal{F}$ is pointwise bounded, i.e. for each $x \in X$ there exists $b_x > 0$ with $\|Tx\| \leq b_x$ for all $T \in \mathcal{F}$. Then $\sup\limits_{T\in\mathcal{F}} \|T| < \infty$.*

Proof. For each $T \in \mathcal{F}$ consider the function $f_T \colon X \to \mathbb{R}_+$ defined by $f_T(x) = \|QTx\|$, $x \in X$, where $Q \colon Y \to Y/\overline{T(0)}$ is the quotient map. Firstly, we show that f_T is lower semicontinuous on $(X, \bar{\rho_q})$. Let $x \in X$ and $(x_n)_n$ be a sequence in X such that $\bar{\rho_q}(x, x_n) \to 0$ if $n \to \infty$. We have

$$f_T(x) - f_T(x_n) = \|QTx\| - \|QTx_n\| \leq \|QTx - QTx_n\| \leq \|T| q(x - x_n) \to 0 \ \text{ if } \ n \to \infty.$$

It follows that for each $\varepsilon > 0$ there is $n_0 \in \mathbb{N}$ such that $f_T(x) - f_T(x_{n_0}) < \varepsilon$. This means that the function f_T is lower semicontinuous on $(X, \bar{\rho_q})$. An application of the previous lemma to the family $\mathcal{A} = \{f_T : T \in \mathcal{F}\}$ reveals the existence of a nonempty open subset U of (X, q) and a $d > 0$ such that $f_T(x) \leq d$ for all $T \in \mathcal{F}$ and $x \in U$. Let B_q be an open ball with center z and radius r contained in U. For $x \in rB_q$, we have $x + z \in B_q$ and

$$\|Tx\| = \|QTx\| \leq \|QT(x+z)\| + \|QT(-z)\| = f_T(x+z) + f_T(-z) \leq d + b_{-z} = b.$$

Therefore

$$\sup_{T\in\mathcal{F}} \|T| \leq \frac{b}{r}.$$

□

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Ми вивчаємо неперервність лінійних відношень, що визначені на асиметричних нормованих просторах із значеннями у нормованих просторах. Ми даємо деяку геометричну характеризацію цих відображень. Як застосування ми доводимо теорему Банаха-Штейнгауза в контексті асиметричних нормованих просторів.

Ключові слова і фрази: лінійне відношення, багатозначний лінійний оператор, асиметрична норма.