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Polynomials on algebraic manifolds

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In this paper, we study polynomials on algebraic subvarieties $A \subset \mathbb{C}^N$, $\dim A = n$, $n \le N$, in the point of view their polynomial extensions into the enclosing space $\mathbb{C}^N$. It is important in observing extremal Green function $V(z, K)$ for compact subset $K \subset A$.

Key words and phrases: algebraic set, polynomial, Green function, polynomial approximation.

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Introduction

A holomorphic function $p(z) \in \mathcal{O}(\mathbb{C}^N)$ given on the space $\mathbb{C}^N$ is called to be a polynomial on an algebraic set $A \subset \mathbb{C}^N$, $\dim A = n$, with respect to $\mathbb{C}^N$, if for all $z \in A$ it holds an inequality $|p(z)| \le C(1 + \|z\|)^d$ with positive constants $C, d > 0$, where $\|z\|$ is the Euclidean norm in $\mathbb{C}^N$. The smallest of the numbers d satisfying this inequality is called degree of the polynomial p, $d = \deg p$. We denote by $\mathcal{P}_A^m(\mathbb{C}^n)$ (or $\mathcal{P}_A^m$ for short) the class of all polynomials p on an algebraic manifold $A \subset \mathbb{C}^N$ with $\deg p \le m$. Let $\mathcal{P}_A = \bigcup_{m=1}^{\infty} \mathcal{P}_A^m$ is the class of all polynomials on A. Obviously, in the definition of polynomials instead of the function $p(z) \in \mathcal{O}(\mathbb{C}^N)$ we can take the functions $p(z) \in \mathcal{O}(A)$, since every holomorphic function on submanifold $A \subset \mathbb{C}^N$, $\dim A = n$, extends to $\mathbb{C}^N$ as a holomorphic function.

Note that the algebraic manifolds are parabolic and the notion of polynomial can be defined by another way, using special exhaustion function $\rho(z)$. For complex manifolds of arbitrary dimensions its parabolicity means the existence of a special exhaustion function (see P. Griffiths and J. King [5], W. Stoll [15,16], A. Sadullaev [10], A. Aytuna and A. Sadullaev [1,2] etc.).

Definition 1. *A Stein manifold $X \subset \mathbb{C}^N$ of dimension n is called S-parabolic manifold, if there exists a special exhaustion function $\rho(z)$ that satisfies the conditions*

a) $\rho(z) \in psh(X)$, i.e. ρ is plurisubharmonic function, and $\{\rho \le M\} \subset\subset X$ for all $M \in \mathbb{R}$;

b) ρ is maximal plurisubharmonic function on the complement of a compact subset $K \subset\subset X$, i.e. $(dd^c\rho)^n = 0$ on $X \setminus K$.

Let us define the concept of polynomials on a S-parabolic manifold $X \subset \mathbb{C}^N$.

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Definition 2. *If for a function $p(z) \in \mathcal{O}(X)$ there exist positive real numbers c, d such that for all $z \in X$ the inequality*

$$\ln |p(z)| \leq d \cdot \rho^+(z) + c \tag{1}$$

holds, where $\rho^+(z) = \max\{0,\ \rho(z)\}$, then the function p is called ρ-polynomial of degree $\leq d$. The minimal value of d in the inequality (1) *is called the degree of the polynomial p.*

In order to determine the special exhaustion function on an algebraic variety $A \subset \mathbb{C}^N$, we will use the well-known geometric criterion for algebraicity by W. Rudin [9]: any algebraic set of pure dimension n, $1 \leq n < N$, after a suitable unitary transformation can be represented in the form

$$A \subset \left\{ z \in \mathbb{C}^N :\ \|z''\| < c(1 + \|z'\|)^\beta \right\},\ z' = (z_1, \ldots, z_n),\ z'' = (z_{n+1}, \ldots, z_N),$$

where c, β are constants. In the work [11], A. Sadullaev pointed out that here we can actually put $\beta = 1$, i.e. an algebraic variety can always be embedded into a cone

$$A \subset \left\{ z \in \mathbb{C}^N :\ \|z''\| < c(1 + \|z'\|) \right\}.$$

Then as a special exhaustion function on A we can take the function $\rho(z) = \ln^+ \|z'\|$, $z \in A$, $\rho(z) \in psh(A)$, $(dd^c \rho(z))^n = 0$ on $A \backslash \{\|z'\| \leq 1\}$.

According to the Definition 2, polynomials on A in terms of a special exhaustion function ρ are defined as follows.

Definition 3. *Assume that for a function $p(z) \in \mathcal{O}(A)$ there exist positive real numbers c, d such that for all $z \in A$ the inequality $\ln |p(z)| \leq d \cdot \ln^+ \|z'\| + c$ holds. Then the function $p(z)$ is called a ρ-polynomial of degree $\leq d$ on algebraic manifold A.*

It is easy to check that if we choose the constant c sufficiently large, then

$$\|z'\| \leq \|z\| \leq 2c\|z'\|$$

on $A \backslash \{\|z'\| \leq 1\}$. Therefore the restriction $p = P|_A$ of an arbitrary polynomial $P(z) \in \mathcal{P}(\mathbb{C}^N)$ is a ρ-polynomial on A.

1 Extension of polynomials from algebraic submanifolds

We are interested on the opposite question: if $p(z) \in \mathcal{O}(A)$ is a polynomial on A, will it be a restriction of some polynomial $P(z)$ in $\mathbb{C}^N$? Positive answer to this question is important for the study of the Green function $V(z, K)$ of a compact subset $K \subset A$, which plays a key role in the questions of polynomial approximations.

The problem of continuation of analytic functions from algebraic subvarieties to the enclosing complex space, with various restrictions on growth, to powers of polynomials, were studied by many famous mathematicians. In the works of L. Hörmander [6], H. Skoda [14], J.E. Björk [3], J.P. Demailly [4], Sh. Nakano [7], Y. Nishimura [8] and other authors such questions are resolved using the theory of $\bar{\partial}$-equations and L^2-estimates.

In our case, it is interesting that for an arbitrary polynomial $p(z)$ on an algebraic variety $A \subset \mathbb{C}^N$ there is a continuation from A to $\mathbb{C}^N$ as a polynomial with a controlled degree.

Example 1. *The graph $A = \{z_{n+1} = g(z)\}$ of a polynomial $g(z)$ in the space $\mathbb{C}^n_z$, $\deg g(z) = s$, is a S-parabolic manifold with the special exhaustion function $\rho(z, z_{n+1}) = \ln^+ \|z\|$, where $(z, z_{n+1}) \in A$, $\rho(z, z_{n+1}) \in psh(A)$, $(dd^c\rho(z, z_{n+1}))^n = 0$ on $A \setminus \{\|z\| \le 1\}$.*

It is clear, that if $p(z)$ is a ρ-polynomial on A of degree d, i.e. if $\ln|p(z)| \le d \cdot \rho^+(z, z_{n+1}) + c$, $z \in A$, then the polynomial $P(z, z_{n+1}) = p(z) \in \mathcal{P}(\mathbb{C}^{n+1})$ satisfies the same property

$$\ln|P(z, z_{n+1})| \le d \cdot \rho^+(z, z_{n+1}) + c \le d \cdot \ln^+ \|z\| + c, \quad (z, z_{n+1}) \in \mathbb{C}^{n+1}.$$

Therefore, each ρ-polynomial extends to $\mathbb{C}^{n+1}$ and this extension will be a polynomial of degree $\le d$.

And vice versa, restriction $p = P|_A$ of a polynomial $P(z, z_{n+1}) \in \mathcal{P}(\mathbb{C}^{n+1})$ is a ρ-polynomial on A, and $\deg P|_A \le \deg P = q$. Indeed, if $(z, z_{n+1}) \in A \setminus \{\|z\| \le 1\}$, we have $\rho^+(z, z_{n+1}) = \rho^+(z, g(z)) = \ln^+ \|z\|$. Consequently, if $P(z, z_{n+1}) = \sum_{k=0}^{q} c_k(z, z_{n+1})$, where $c_k(z, z_{n+1})$ are homogeneous polynomials of degree $\deg c_k(z, z_{n+1}) \le q$, then for some $C > 0$ we have

$$|P(z, t(z))| \le \sum_{k=0}^{q} |c_k(z, t(z))| \le C \cdot \sum_{k=0}^{q} \|z\|^q = C(q+1) \cdot \|z\|^q.$$

Hence, $\ln|P(z, g(z))| \le q \cdot \rho^+(z) + c$, with $c = \ln C(q+1)$ and the restriction $P|_A = P(z, t(z))$ is a ρ-polynomial of the same degree q as polynomial $P(z, z_{n+1}) \in \mathcal{P}(\mathbb{C}^{n+1})$.

In the general case for arbitrary algebraic manifolds, we have the following result.

Theorem 1 (see J. Bjork [3]). *For a fixed algebraic submanifold $A \subset \mathbb{C}^N$ there exists a constant $b(A)$, depending only on submanifold A, such that for each polynomial $p(z)$ on A there exists a polynomial $P(z)$ in $\mathbb{C}^N$ satisfying $P|_A = p$, $\deg P \le \deg p + b(A)$.*

2 Extremal functions

The classical Bernstein-Walsh theorem establishes a close connection between rate of polynomial approximation of a function $f(z)$, defined on a compact set $K \subset \mathbb{C}$, and its holomorphic extension to a neighborhood of K, defined by well-known Green function $V(z, K)$. In 1962, J. Siciak [13] proved the following generalization of this theorem to the multidimensional case. Let $K \subset \mathbb{C}^n$ be a regular compact set and $e_d(f, K) = \inf_{P \in \mathcal{P}^d(\mathbb{C}^n)} \|f(z) - P(z)\|_K$ is a minimal deviation of a function $f \in C(K)$ from the class of polynomials $\mathcal{P}^d(\mathbb{C}^n)$ on K. A function $f(z)$ initially defined on a compact set K extends holomorphically into a neighborhood $D_R = \{z \in X : \Phi(z, K) < R\}$, $R > 1$, if and only if the inequality $\overline{\lim}_{d\to\infty} e_d^{1/d}(f, K) \le 1/R$ holds. Here $\Phi(z, K) = \sup\{|p(z)|^{1/\deg p} : \ p \in \mathcal{P}(\mathbb{C}^n), \ \|p\|_K \le 1\}$ is Siciak extremal function of compact set K.

Let us make a reservation right away that the extremal function $\Phi(z, K)$ is very inconvenient from the point of view of its study, due to its algebraic definition using polynomials. In this regard, the Green function $V(z, K)$, which is defined by plurisubharmonic functions, is much more convenient. The last one is simpler to define and in the study of its geometric properties, local properties of plurisubharmonic functions are usually used.

In the space $\mathbb{C}^N$ it holds the equality (see [12])

$$V(z, K) \equiv \ln \Phi(z, K), \tag{2}$$

which plays a key role in the multidimensional Bernstein-Walsh theorem. On an algebraic variety $A \subset \mathbb{C}^N$, $K \subset\subset A$, the Green function is defined as

$$V_A^*(z,K) = \overline{\lim_{w \to z}} V_A(w,K),$$

where $V_A(z,K) = \sup\{u(z) \in psh(A) : \ u(z) \le c_u + \rho(z), \ u|_K \le 0\}$.

The following result holds (it was proved together with A. Sadullaev).

Theorem 2. *For any compact set $K \subset A$ the following equality holds*

$$V_A(z,K) = \ln \Phi_A(z,K),$$

where $\Phi_A(z,K) = \sup\{|p(z)|^{1/\deg p} : \ p(z) \in \mathcal{P}_A, \ |p(z)|_K \le 1\}$.

Note that equality (2) in the space $\mathbb{C}^n$ was proved using embedding $\mathbb{C}^n \subset P^n$ and using homogeneous polynomials in P^n, so we cannot use such a method to prove a similar result on an algebraic variety A.

Proof. If we, in defining a function $\Phi_A(z,K)$ for a fixed $t \in \mathbb{N}$, limit ourselves to only polynomials of degree $\ge t$, i.e.

$$\Phi_A^t(z,K) = \sup\{|p(z)|^{1/\deg p} : \ p(z) \in \mathcal{P}_A, \ |p(z)|_K \le 1, \ \deg p \ge t\},$$

then $\Phi_A(z,K) = \Phi_A^t(z,K)$.

Indeed, it is clear that $\Phi_A^t(z,K) \le \Phi_A(z,K)$. On the other hand, for any fixed $\varepsilon > 0$ and any fixed point $z^0 \in A$ there is a polynomial $p(z) \in \mathcal{P}_A$, such that $|p(z^0)|^{1/\deg p} \ge \Phi_A(z^0,K) - \varepsilon$, because of $\Phi_A(z,K) = \sup\{|p(z)|^{1/\deg p} : \ p(z) \in \mathcal{P}_A, \ |p(z)|_K \le 1\}$. Raising the polynomial $p(z) \in \mathcal{P}_A$ to a power, we also preserve this relation

$$|p^j(z^0)|^{1/j \deg p} \ge \Phi_A(z^0,K) - \varepsilon, \quad \deg p^j(z) = j \deg p.$$

If we take j so large that $j \deg p \ge t$, we get

$$\begin{aligned} \Phi_A^t(z^0,K) &= \sup\{|p(z^0)|^{1/\deg p} : \ p(z) \in \mathcal{P}_A, \ |p(z)|_K \le 1, \ \deg p \ge t\} \\ &\ge |p^j(z^0)|^{1/j \deg p} \ge \Phi_A(z^0,K) - \varepsilon. \end{aligned}$$

Since the number $\varepsilon > 0$ and the point $z^0 \in \mathbb{C}^n$ are arbitrary, we get $\Phi_A^t(z,K) \ge \Phi_A(z,K)$ and therefore

$$\Phi_A^t(z,K) \equiv \Phi_A(z,K) \ \ \forall \ t \in \mathbb{N}.$$

Now we show that for any compact $K \subset A$ the equality $\Phi_A(z,K) \equiv \Phi(z,K)$, $z \in A$, holds. In fact, if we take a polynomial $p(z)$ in $\mathbb{C}^N$, then its restriction $q(z) = p(z)|_A$ is a polynomial of the degree less than or equal to $\deg p$. Therefore,

$$\begin{aligned} \Phi(z,K) &= \sup\{|p(z)|^{1/\deg p} : \ p \in \mathcal{P}(\mathbb{C}^N), \ |p(z)|_K \le 1\} \\ &\le \sup\{|q(z)|^{1/\deg q} : \ q \in \mathcal{P}_A, \ |q(z)|_K \le 1\} = \Phi_A(z,K), \quad z \in A. \end{aligned} \tag{3}$$

On the other hand, for an arbitrary polynomial $q(z)$ on an algebraic variety $A \subset \mathbb{C}^N$ there is a continuation $q(z)$ from A to $\mathbb{C}^N$ as a polynomial of a controlled degree. More precisely, there is a constant $b(A)$ depending only on A such that for any polynomial $q(z)$ on A there is such a polynomial $p_q(z)$ in $\mathbb{C}^N$ with $p_q|_A = q$, $\deg p_q = \deg q + b(A)$ (Theorem 1). Hence for $z \in A$ we have

$$\begin{aligned}\Phi_A(z,K) &= \sup\{|q(z)|^{1/\deg q} : \ q \in \mathcal{P}_A, \ |q(z)|_K \le 1\} \\ &= \sup\{|q(z)|^{1/\deg q} : \ q \in \mathcal{P}_A, \ |q(z)|_K \le 1, \ \deg q \ge t\} \\ &= \sup\{|p_q(z)|^{1/\deg q} : \ p_q \in \mathcal{P}(\mathbb{C}^N), \ \ |p_q(z)|_K \le 1, \ t \le \deg p_q \le \deg q + b(A)\}.\end{aligned}$$

But when $|p_q(z)| > 1$, the inequality

$$|p_q(z)|^{1/\deg q} \le |p_q(z)|^{1/(\deg p_q - b(A))} = (|p_q(z)|^{1/\deg p_q})^{\deg p_q/(\deg p_q - b(A))}$$

holds.

Note, that for each given $\varepsilon > 0$ for a large enough $t \in \mathbb{N}$ we have ratio

$$\frac{\deg p}{\deg p - b(A)} = \frac{1}{1 - b(A)/t} = 1 + \varepsilon(t), \quad \varepsilon(t) \to 0 \quad \text{as} \quad t \to \infty.$$

Therefore, when $|p_q(z)| > 1$, the inequality

$$\begin{aligned}|p_q(z)|^{1/\deg q} &\le (|p_q(z)|^{1/\deg p_q})^{\deg p_q/(\deg p_q - b(A))} \\ &= |p_q(z)|^{1/\deg p_q} \cdot (|p_q(z)|^{1/\deg p_q})^{\varepsilon(t)} = |p_q(z)|^{1/\deg p_q} \cdot M^{\varepsilon(t)}\end{aligned}$$

holds, where $M = |p_q(z)|^{1/\deg p_q} \le \Phi(z,K)$, $z \in A$.

But if $|p_q(z)| > 1$, we have $|p_q(z)|^{1/\deg q} \le |p_q(z)|^{1/\deg p_q}$ because of $\deg p_q \ge \deg q$. Hence,

$$\begin{aligned}\Phi_A(z,K) &= \sup\left\{|p_q(z)|^{1/\deg q} : \ p_q \in \mathcal{P}(\mathbb{C}^N), \ |p_q(z)|_K \le 1, \ t \le \deg p_q \le \deg q + b(A)\right\} \\ &\le \sup\left\{|p_q(z)|^{\frac{1}{\deg p_q}} (\max 1, M)^{\varepsilon(t)} : p_q \in \mathcal{P}(\mathbb{C}^N), |p_q(z)|_K \le 1, t \le \deg p_q \le \deg q + b(A)\right\} \\ &= \left(\max\{1, \Phi(z,K)\}\right)^{\varepsilon(t)} \sup\left\{|p_q(z)|^{1/\deg p_q} : \ p_q \in \mathcal{P}(\mathbb{C}^N), \ |p_q(z)|_K \le 1, \ t \le \deg p_q\right\} \\ &= (\max\{1, \Phi(z,K)\})^{\varepsilon(t)} \cdot \Phi(z,K).\end{aligned}$$

Thus, when $t \to \infty$ we have $\Phi_A(z,K) \le \Phi(z,K), z \in A$, and this together with inequality (3) gives $\Phi_A(z,K) = \Phi(z,K)$, $z \in A$. Since $\ln \Phi(z,K) \equiv V(z,K)$ for any compact subset $K \subset \mathbb{C}^N$, then $\ln \Phi_A(z,K) = V(z,K)$. Since for any function from the Lelong class $L(K) = \{u(z) \in psh(\mathbb{C}^N) : \ u(z)|_K \le 0, \ u(z) \le c_u + \ln^+ |z|\}$ its restriction to the algebraic submanifold $A \subset \mathbb{C}^N$ belongs to the Lelong class $L_A(K) = \{u(z) \in psh(A) : \ u(z)|_K \le 0, \ u(z) \le c_u + \rho(z)\}$, we get the inequality $V(z,K) \ge V_A(z,K)$. Moreover, for any polynomial $p \in \mathcal{P}_A$ it holds the Bernstein-Walsh inequality (see [1])

$$\ln \left[\frac{|p(z)|}{\|p(z)\|_K}\right]^{1/\deg p} \le V_A(z,K), \ z \in A,$$

since $\ln\left[|p(z)|/\|p(z)\|_K\right]^{1/\deg p} \in L_A$, and this implies

$$\begin{aligned}V_A(z,K) &= \sup\left\{u(z) \in psh(A) : \ u(z)|_K \le 0, \ u(z) \le c_u + \rho(z)\right\} \\ &\ge \sup\left\{\ln |p(z)|^{1/\deg p} : \ p \in \mathcal{P}_A, \ |p(z)|_K \le 1\right\} = \ln \Phi_A(z,K).\end{aligned}$$

Now, we can complete the proof of the property $V_A(z,K) = \ln \Phi_A(z,K)$, $z \in A$. Namely, the last equality follows from the chain of relations $V_A(z,K) \ge \ln \Phi_A(z,K) = \ln \Phi(z,K) = V(z,K) \ge V_A(z,K)$. $\square$

References

[1] Aytuna A., Sadullaev A. Polynomials on parabolic manifolds. In: Ibragimov Z., Levenberg N., Pinchuk S., Sadullaev A. (Eds.) Topics in several complex variables. First USA-Uzbekistan Conference on Analysis and Mathematical Physics, May 20–23, 2014, California State University, Fullerton, CA. Contemporary Mathematics, 662. AMS, Providence, RI, 2016, 1–22.

[2] Aytuna A., Sadullaev A. *Parabolic Stein manifolds*. Math. Scand. 2014, **114** (1), 86–109. doi:10.7146/math.scand.a-16640

[3] Björk J.E. *On extensions of holomorphic functions satisfying a polynomial growth condition on algebraic varieties in* $\mathbb{C}^n$. Ann. Inst. Fourier 1974, **24** (4), 157–165. doi:10.5802/aif.535

[4] Demailly J.P. Scindage holomorphe d'un morphisme de fibrés vectoriels semi-positifs avec estimations L^2. In: Lelong P., Skoda H. (Eds.) Séminaire Pierre Lelong-Henri Skoda (Analyse) Années 1980/81. Lecture Notes in Mathematics, 919. Springer, Berlin, Heidelberg, 1982, 77–107. doi:10.1007/BFb0097044

[5] Griffiths P., King J. *Nevanlinna theory and holomorphic mappings between algebraic varieties*. Acta Math. 1973, **130**, 145–220. doi:10.1007/BF02392265

[6] Hörmander L. *L^2 estimates and existence theorems for the $\bar{\partial}$-operator*. Acta Math. 1965, **113**, 89–152. doi:10.1007/BF02391775

[7] Nakano Sh. *Extension of holomorphic functions with growth conditions*. Publ. Res. Inst. Math. Sci. 1986, **22** (2), 247–258. doi:10.2977/prims/1195178068

[8] Nishimura Y. *Problème d'extension dans la théorie des fonctions entières d'ordre fini*. J. Math. Kyoto Univ. 1980, **20** (4), 635–650. doi:10.1215/kjm/1250522163 (in French)

[9] Rudin W., *A geometric criterion for algebraic varieties*. J. Math. Mech. 1968, **17** (7), 671–683.

[10] Sadullaev A. *Deficient divisors in the Valiron sense*. Math. USSR Sb. 1980, **36** (4), 535–547. doi:10.1070/SM1980v036n04ABEH001861

[11] Sadullaev A. Criteria for analytic sets to be algebraic. In: Holomorphic functions of several complex variables. Institute of Physics SB AS USSR, Krasnoyarsk, 1976, 107–122. (in Russian)

[12] Sadullaev A. *Plurisubharmonic functions*. In: Khenkin G.M., Vitushkin A.G. (Eds.) Several Complex Variables II. Encyclopaedia of Mathemtical Sciences, 8. Springer, Berlin, Heidelberg, 1994, 59–106. doi:10.1007/978-3-642-57882-3_2

[13] Siciak J. *On some extremal functions and their applications in the theory of analytic functions of several complex variables*. Trans. Amer. Math. Soc. 1962, **105**, 322–357. doi:10.1090/S0002-9947-1962-0143946-5

[14] Skoda H. *Application des techniques L^2 à la théorie des idéaux d'une algèbre de fonctions holomorphes avec poids*. Ann. Sci. de l'École Norm. Sup. 1972, **5** (4), 545–579. (in French)

[15] Stoll W. Value distribution on parabolic spaces. Lecture Notes in Mathematics, 600. Springer, Berlin, Heidelberg, 1977.

[16] Stoll W. *The characterization of strictly parabolic manifolds*. Ann. Scuola Norm. Super. Pisa-Classe Sci. 1980, **7** (1), 87–154.

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У цій статті ми досліджуємо поліноми від алгебраїчних підмноговидів $A \subset \mathbb{C}^N$, $\dim A = n$, $n \leq N$, з точки зору їх поліноміальних розширень в охоплюючий простір $\mathbb{C}^N$. Це важливо для спостереження екстремальної функції Гріна $V(z, K)$ для компактної підмножини $K \subset A$.

Ключові слова і фрази: алгебраїчна множина, поліном, функція Гріна, поліноміальна апроксимація.