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THE GENERALIZATION OF THE COMPARATIVE ADVANTAGE THEORY

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Abstract. Economic textbooks relate the principle of comparative advantage using examples of two products and two countries, the *2x2* case. We shall suggest an approach describing any finite number of products *m* and countries *n*, the *mxn* case, where *m*>2, *n*>2. For this purpose the linear programming will be used.

Keywords: comparative advantage, absolute advantage, linear programming, intermediate goods, technological coefficient.

1. INTRODUCTION

The idea of comparative advantage was suggested by David Ricardo at the beginning of the 19th century. It refers to the situation when a given country or region is less efficient in production of all goods and services as compared to what others producers do. The lack of absolute advantage in case of any product is not an obstacle for this area to find its place in the world economy. Even under the assymetry in cost efficiency the production of goods and services should be shared between the less profitable area and the rest of the world, and this will arrive to the benefit of all sides concerned. A country, a region or a firm can have no absolute advantage in case of any product but it has a comparative advantage in delivering some products.

How one can identify comparative advantages of a given entity? To find them one must take into account the relation of production costs, which are usually understood as labour inputs, between different producers. Economic textbooks relate this principle using examples in which it is customary to discuss a world economy consisting of two products and two countries (Brakman, Garretsen, van Marrewijk, van Witteloostuijn 2006, Carbaugh 2009, Krugman 2010, Salvatore 2011). We shall call this a 2x2 case. It is not easy to generalize these examples and to have a theory describing any finite number of products *m* and countries *n*, which we shall call a *mxn* case. So long there is no presentation of the principle in the generalized *nxn* form. The *2x2* case originates form the Ricardo's text and until now was not successfully transformed to the *mxn* case, where *m*>2, *n*>2. The state of theory, as it commonly lectured, is that one must believe that the *mxn* case is real, very similar to the *2x2* case, is in a way a generalization of the *2x2* case, but it is not clear how precisely this transformation should be done. Some place is left to the intuition.

A big effort was put to solve the *nxn* problem. A model of *m* products, *n* countries, intermediate goods and a choice of production techniques was suggested by Shiozawa (2007). Deardorff (2005) has included in the model trade costs of producing and delivering goods and services. Cassey (2012) changed Deardorff's result by agglomeration of exporters from the point of view of destination of shipments. My paper is one more contribution to solve the Ricardian problem of *m* products, *n* countries and wused factors of production, where $min(m,n,w)>2$. I shall call it a *mxnxw* model. I go back to the original question of assignment products to countries. It is possible to consider just one factor of production, but it is not difficult to assume that one has more than one factor.

2. Example. A Classical Approach

Let us start with an example illustrating the comparative advantage principle. It will help us to understand the new approach to be described later.

Example 1. The world economy consists of two countries *A* and *B*, two products are manufactured, that is shirts and computers. The measure of productivity in both countries is the number of manufactured products per hour. We assume in Table 1 that country *A* can produce 6 shirts or 4 computers in an hour.

It is easily seen that *A* has absolute advantage over *B* in both products. Does it pay for yhis country to trade with country *B*? The intuition points that *A* can possibly deal with shirts and country *B* with computers, is it really so?

	Country A	Country B
Shirts	6	1
Computers	4	2

Tab. 1. Production per hour.
Source: Example prepared by the author.

Country *A* produces shirts 6 times more efficiently than country *B*, for computers the efficiency rate is 2 (4 divided by 2). This gives country *A* a comparative advantage over country *B* in shirts. Country *B* notes this kind of advantage in computers, because its weakness in computers is just 1/2, not 1/6 as for shirts.

Let us assume that country *A* can use 4 working hours, which will be equally allocated for shirts and computers. Country *B* can use 12 working hours to be shared 8:4 between shirts and computers. The world production before specialization is in the last column of Table 2.

	Country A (4 hours)	Country B (12 hours)	World production
Shirts	12	8	20
Computers	8	8	16

Tab. 2. Production before trade.
Source: Example prepared by the author.

As suggested country *A* will keep on delivering shirts, country *B* computers, what is in line with the comparative advantage principle. The new arrangement of world production is given in Table 3.

	Country *A*	Country *B*	World production
Shirts	24	0	24
Computers	0	24	24

Tab. 3. Production in line with comparative advantage.
Source: Example prepared by the author.

The specialization and the division of labour made the global output bigger what is advantageous for both countries.

3. COMPARATIVE ADVANTAGES, *2x2* AND *nxn* CASES

We shall make the following assumptions. There is only one production factor. The labour is perfectly mobile between economic sectors but not between countries. The labour productivity remains unchanged for every level of production.

Let us start with *2x2 model* (Winters 1991). The world economy contains two countries A and B and two products C and F. The input necessary to manufacture product i in country j is l_j^i, $i=A,B$, $j=C,F$. The principle of comparative advantage points to producing product F in country A if and only if

$$\frac{l_F^A}{l_F^B} \prec \frac{l_C^A}{l_C^B} \tag{1}$$

or, to put it the other way, if

$$\frac{l_F^A}{l_C^A} \prec \frac{l_F^B}{l_C^B}. \tag{2}$$

There is some simplification in calling the *2x2* model a case of two products and two countries. In fact this is a *2x2x1 model*, because we can identify one more element, being the labour as a production factor.

Generally, is it possible to have a *IxKxJ model*, where I, J and K accordingly are the number of products, the number of factors and the number of countries. Following (1) and (2) below in (3) the number of countries is $K=2$ and the number of products I is just finite, in (4) K is any natural number and $I=2$. In (3) and (4) a single factor $J=1$ occurs

$$\frac{l_1^A}{l_1^B} \le \frac{l_2^A}{l_2^B} \le \ldots \le \frac{l_I^A}{l_I^B}, \tag{3}$$

$$\frac{l_F^1}{l_C^1} \le \frac{l_F^2}{l_C^2} \le \ldots \le \frac{l_F^K}{l_C^K}. \tag{4}$$

In both sequences (3) and (4) we must find cutoff points. This can be done provided we get an extra criterion to find such a point. If we do not have one there is no way to divide production between countries A and B. The question is: which member of sequence (3) is to be attributed to country A, which one is to be left to country B? The same problem is with cutting the sequence (4).

The cases of 2 countries and any number of products were studied by G. Haberler (1933) and J. Viner (1937). A model of aggregate global production with many products and countries was published by F. D. Graham, who suggested a set of equations and inequalities to be solved with the trial and error method (Graham 1948). L. W. McKenzie has extended this model by introducing an efficiency criterion method (McKenzie 1953, 1955). R. W. Jones was allocating production to countries when the numbers of countries and products are equal [Jones 1961]. He did not change the assumption on a single production factor used what lead him to a table similar to that in the input-output analysis. The *2xn* and *mx2* cases are subject of study (Dornbusch, Fischer, Samuelson, 1976, 1980). The *mxn* case, where $m>2$, $n>2$, has no solution in every case.

4. COMPARATIVE ADVANTAGES AND THE LINEAR PROGRAMMING

Now we shall construct a *mxnxw* model, where $min(m,n,w)>2$, a generalized case of the comparative advantages problem. Let x_{ik} be the production level of product i in country k, and b_{jk} the stock of factor j in country k, $i=1,\ldots,I$, $j=1,\ldots J$, $k=1,\ldots,K$. The coefficient a_{ijk} stands for the input of factor j necessary in country k to produce a unit of product i. These coefficients can be called technological. All inputs (factor levels) and outputs (production levels) are measured in money terms what makes it possible to compare them.

We are looking for a production plan for all countries which will make the world output maximal. For this purpose we suggest the following linear programming problem

$$\sum_{i=1}^{I}\sum_{k=1}^{K} x_{ik} \to \max \tag{5}$$

subject to conditions

$$\sum_{i=1}^{I} a_{ijk}x_{ik} \le b_{jk},\ j=1,\ldots,J,\ k=1,\ldots,K, \tag{6}$$

$$x_{ik} \ge 0,\ i=1,\ldots,I,\ k=1,\ldots,K. \tag{7}$$

It is possible that inputs of factors are divided into two parts. The first one remains at disposal of countries concerned, the second part is available to all countries because it is a subject of free international trade. This differentiation to be noticed when constructing our model. We assume that factors $1,\ldots,J'$ are located in particular countries and cannot be transferred (sold) abroad, other factors $J'+1,\ldots,J$ can be traded and used in any country. Now we substitute a set of constraints (6) by two groups of constraints

$$\sum_{i=1}^{I} a_{ijk}x_{ik} \le b_{jk},\ j=1,\ldots,J',\ k=1,\ldots,K, \tag{8}$$

$$\sum_{i=1}^{I}\sum_{k=1}^{K} a_{ijk}x_{ik} \le b_j,\ j=J'+1,\ldots,J. \tag{9}$$

So long we have introduced three formulations of the comparative advantage problem. Firstly, all factors of production are distributed between countries, this is the problem (5), (6), (7). A given factor is included in this very model many times, always as a stock at disposal of a particular country. Secondly, some factors are distributed among countries and some of them are available for all countries when freely trading. This is a linear programming problem (5), (7), (8), (9). And thirdly, the access to every factor is unlimited, that means the total stock of all factors is to be shared by all participants. This is the problem (5), (7), (9), when $J'=0$. In the third case the number of local deposits of factors and the number of factors used are equal.

These three economic situations we will call respectively P, Q and R. These are three varieties of the $m x n x w$ model, where $min(m,n,w)>2$. In any of these problems the number of variables is IxK. In P the number of constraints is a product: number of factors by number of countries. In Q we have $J'xK+(J-J')$ constraints, the sum of products: number of divided factors by number of countries plus number of factors of unlimited access. In the latter case R the number of constraints is the number of factors.

Practically the case Q is dominant. Situations P and R are from the economic point of view rarely met.

In case P the linear programming problem (5), (6), (7) can be substituted by a set of smaller problems. Instead of a single big problem of IxK variables and JxK constraints we can have K problems of I variables and J constraints. For every $k=1,\ldots,K$ we obtain a smaller problem

$$\sum_{i=1}^{I} x_{ik} \to \max \tag{10}$$

subject to

$$\sum_{i=1}^{I} a_{ijk} x_{ik} \le b_{jk}, \; j = 1,..., J, \tag{11}$$

$$x_{ik} \ge 0, \; i = 1,..., I. \tag{12}$$

The decomposition of the starting problem *P* is not just a matter of computing technique. It helps us to interpret the starting problem. If factors are fully distributed among countries the global production maximization is simply maximizing of local production in every country. In order to have this no universal coordination is needed. Every entity is well equipped to do its best and this is to the benefit of all. The optimal plan for country *k* does not depend on what is going on abroad.

The problem *R* refers to global availability of resources. This makes it possible to concentrate the production in a single country, what means that some countries will deliver nothing. It is also possible to produce only a group of products included on the starting list, that is to resign from producing some goods and services. To maximize the objective function and to utilize fully resources one should keep on manufacturing certain products, not of all them. Paradoxically, this effect does not meet expectations of the comparative advantage principle, which are to some extent tacit, but really exist. The expectations are: we want to produce all *I* goods and services from the list (what is a tacit expectation) and we want to locate some manufacturing in any country (what is not a tacit expectation).

If the number of factors *J* is much smaller than the number variables *IxK*, only some elements of the optimal solution of problem (5), (7), (9) will be different from zero. For instance, if there are two factors *J=2*, three countries *K=3* and three products *I=3*, only two variables will not equal to zero. Consequently, the production of some merchandise will begin and the production of two other goods and services can be located in one country.

All this makes us conclude that in practice the problem *Q* is the most important one and worth analyzing. The problems *P* and *R* are to be mentioned because to make the theory complete.

One can ask who is to solve a linear programming problem to find out who is to produce goods and services. Both cases, the classical ones, (1) or (2), and just described ones, (5), (7), (9), are to be handled by Adam Smith's "invisible hand" the market, that is by the self-regulating economic mechanism. The world economy is a system of great many individual actions which are in a certain way harmonized. This harmony is to be observed in case of domestic economy and also on the world market.

5. Absolute and Comparative Advantages in the Generalized Model

How to realize what are the advantages of a particular country or region? Are they absolute or comparative ones? How to answer these questions in the real world which is much more complicated than *2x2x1* model prepared for academic purposes? In practice we always have many products, factors and countries, what is the case of theoretical and practical difficulties in monitoring the global economic system. How the answers to these questions are changed if we adopt the linear programming approach?

To define absolute advantages we need a matrix of technological coefficients a_{ijk}, to define comparative advantages we need the optimal solution x_{ik} of the linear programming problem which is to find arrangements in the international division of labour.

We shall say that country *k** has an *absolute advantage* in manufacturing product *i*, if the sum of costs of factors used to prepare this product is minimal

$$\sum_{j=1}^{J} a_{ijk^*} = \min_{1 \le k \le K} \sum_{j=1}^{J} a_{ijk}. \tag{13}$$

Definition (13) does not change the position in this point suggested by Adam Smith.

We shall say that country k^{**} has a *comparative advantage* in delivering product i, if the optimal solution of the linear programming problem, being the generalized model of international division of labour, the following equality occurs

$$x_{ik^{**}} \succ 0. \tag{14}$$

The linear programming approach makes it possible to find precise definitions of commonly used terms what was not easy to have on the ground of traditional absolute and comparative advantage theory. These formal definitions can be also transferred to popular economic discussions and introduce, let it be, pragmatic definitions. We can just say that to get a comparative advantage a country must keep on delivering a product for the long time. This is to happen when the market is not heavily protected. A country enjoys an absolute advantage provided its production costs are really the lowest worldwide what is to be found out after examination of technological coefficients.

6. Examples. A Generalized Approach

A number of examples will illustrate the *mxnxw* model, where *min(m,n,w)>2*. To have a linear programming problem we prepare the following information. We need a matrix of technological coefficients A, with elements of 3 indices, and vectors x, b of variable and resources. The problem is:

$$c^T x \rightarrow \max, \quad Ax \leq b, \; x \geq 0.$$

Rows of this matrix stand for factors, groups of columns for countries and particular columns inside a group for products. Factors are C1, C2 etc., countries K1, K2 etc. and products T1, T2 etc. Matrix A is

	K1				K2				K3			
	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4
C1												
C2												
C3												
C4												

Vectors x and b are

K1				K2				K3			
T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4

C1	C2	C3	C4

Example 2. The first example, a *4x3x2* model, will illustrate the case Q. There are 4 products $I=4$, 2 factors $J=2$ and 3 countries $K=3$

$$I=4,\ J=2,\ J'=1,\ K=3,$$

what means that a single factor is available to all partners. Table 4 includes technological coefficients for the factor 1, used all over the world. To manufacture a unit of product 1 country 1 one needs 8 units of the first factor, in country 2 one needs 10 units, in country 3 one needs 12 units. The last row shows how this factor was distributed among countries. Table 5 includes coefficients for factor 2, its stock is open to trade.

Factor 1	Country 1	Country 2	Country 3
Product 1	8	10	12
Product 2	4	5	7
Product 3	5	7	8
Product 4	2	4	5
Resources in countries	800	700	600

Tab. 4. Technological coefficients for the factor distributed between countries.
Source: Example prepared by the author.

Factor 2	Country 1	Country 2	Country 3	World
Product 1	2	3	5	-
Product 2	3	4	6	-
Product 3	1	2	7	-
Product 4	2	3	6	-
Resources worldwide	0	0	0	1000

Tab. 5. Technological coefficients for the factor distributed between countries.
Source: Example prepared by the author.

The technological coefficients matrix and the transposed resources vector are:

$$\begin{vmatrix} 8 & 4 & 5 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 5 & 7 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 7 & 8 & 5 \\ 2 & 3 & 1 & 2 & 3 & 4 & 2 & 3 & 5 & 6 & 7 & 6 \end{vmatrix}$$

$$\begin{vmatrix} 800 & 700 & 600 & 1000 \end{vmatrix}$$

To construct an objective function we need a unit vector.

The linear programming problem we look for is: maximize the function

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{12} + x_{22} + x_{32} + x_{42} + x_{13} + x_{23} + x_{33} + x_{43}$$

subject to constraints

$8\,x_{11} + 4\,x_{21} + 5\,x_{31} + 2\,x_{41} = 800$

$10\,x_{12} + 5\,x_{22} + 7\,x_{32} + 4\,x_{42} = 700$

$12\,x_{13} + 7\,x_{23} + 8\,x_{33} + 5\,x_{43} = 600$

$2\,x_{11} + 3\,x_{21} + x_{31} + 2\,x_{41} + 3\,x_{12} + 4\,x_{22} + 2\,x_{32} + 3\,x_{42} + 5\,x_{13} + 6\,x_{23} + 7\,x_{33} + 6\,x_{43} = 1000$

$x_{ik} \geq 0$, for $i=1,\ldots,4$; $k=1,2,3$.

The optimal solution is as follows, we omit variables equal to zero

$$x_{31} = 62{,}5;\; x_{41} = 243{,}75;\; x_{32} = 100;\; x_{13} = 50.$$

The maximal value of the objective function is 456,25.

Country 1 should manufacture products 3 and 4, country 2 product 3, and country 3 product 1. These are comparative advantages of three countries.

Countries	Advantages
Country 1	Products 3 and 4
Country 2	Product 3
Country 3	Product 1

Tab. 6. Comparative advantages of the countries.
Source: Example prepared by the author.

As defined in (13), after summing up technological coefficients, absolute advantages of countries are given in table 7.

Countries \ Products	Product 1	Product 2	Product 3	Product 4	Advantages
Country 1	8+2	4+3	5+1	2+2	Products 1, 2, 3, 4
Country 2	10+3	5+4	7+2	4+3	None
Country 3	12+5	7+6	8+7	5+6	None

Tab. 7. Absolute advantages of the countries.
Source: Example prepared by the author.

Two kinds of advantages are different from each other.

Example 3. We have 4 products, $I=4$, and 4 countries, $K=4$. We have also 4 factors locally placed and 3 factors of unlimited access. This makes the LP problem of 16 variables and 7 constraints which is a $4x4x4$ model. Instead of what we had in the previous example equations will substitute inequalities. This is a Q case once again.

We maximize the function

$$x_{11}+x_{21}+x_{31}+x_{41}+x_{12}+x_{22}+x_{32}+x_{42}+x_{13}+x_{23}+x_{33}+x_{43}$$

subject to constraints

$6x_{11}+7x_{21}+x_{31}+3x_{41} \le 600$

$x_{12}+4x_{22}+2x_{32}+4x_{42} \le 600$

$9x_{13}+5x_{23}+x_{33}+6x_{43} \le 600$

$8x_{14}+7x_{24}+3x_{34}+6x_{44} \le 600$

$7x_{11}+6x_{21}+2x_{31}+3x_{41}+8x_{12}+6x_{22}+5x_{32}+x_{42}+2x_{13}+3x_{23}+9x_{33}+7x_{43}+6x_{14}+10x_{24}+2x_{34}+8x_{44} \le 5000$

$5x_{11}+6x_{21}+x_{31}+4x_{41}+9x_{12}+7x_{22}+4x_{32}+2x_{42}+3x_{13}+4x_{23}+8x_{33}+7x_{43}+7x_{14}+9x_{24}+3x_{34}+7x_{44} \le 5000$

$6x_{11}+5x_{21}+x_{31}+3x_{41}+8x_{12}+6x_{22}+3x_{32}+x_{42}+4x_{13}+3x_{23}+9x_{33}+6x_{43}+8x_{14}+8x_{24}+2x_{34}+6x_{44} \le 5000$

$x_{ik} \ge 0$, for $i=1,\dots,4$; $k=1,\dots 4$.

The optimal solution is (only positive variables)

$$x_{31}=600,\ x_{32}=300,\ x_{23}=80{,}159,\ x_{33}=199{,}206,\ x_{34}=133{,}333.$$

Two kinds of advantages are

Countries	Advantages
Country 1	Product 3
Country 2	Product 3
Country 3	Products 2 and 3
Country 4	Product 3

Tab. 8. Comparative advantages of the countries.
Source: Example prepared by the author.

Countries \ Products	Country 1	Country 2	Country 3	Country 4	Advantages
Country 1	6+7+5+6	8+8+9+8	9+2+3+4	8+6+7+8	None
Country 2	7+6+6+5	4+6+7+6	5+3+4+3	7+10+9+8	Product 3
Country 3	1+2+1+1	2+5+4+3	1+9+8+9	3+9+8+9	Product 1
Country 4	3+3+4+3	4+1+2+1	6+7+7+6	5+7+7+6	Products2, 4

Tab. 9. Absolute advantages of the countries.
Source: Example prepared by the author.

Once again two kinds of advantages are different form each other.

Example 4. This is a R case, (5), (7), (9), (16) and (17) model, where the access to all factors is out of any limitations. There are 3 products *I=3*, 2 factors *J=2* and 3 countries *K=3*. The LP problem, a *3x3x2* model, has 2 constraints and 9 variables. The technological coefficients and resources are in table 10.

	Factor 1			Factor 2		
	Country 1	Country 2	Country 3	Country 1	Country 2	Country 3
Product 1	8	10	12	1	2	4
Product 2	4	5	7	3	3	8
Product 3	1	7	8	5	2	4
All resources	800			700		

Tab. 10. Technological coefficients and resources.
Source: Example prepared by the author.

The LP problem we look for is

$$x_{11} + x_{21} + x_{31} + x_{12} + x_{22} + x_{32} + x_{13} + x_{23} + x_{33}$$

subject to constraints

$8\,x_{11} + 4\,x_{21} + x_{31} + 10\,x_{12} + 5\,x_{22} + 7\,x_{32} + 12\,x_{13} + 7\,x_{23} + 8\,x_{33} \le 800$

$x_{11} + 3\,x_{21} + 5\,x_{31} + 2\,x_{12} + 3\,x_{22} + 2\,x_{32} + 4\,x_{13} + 8\,x_{23} + 4\,x_{33} \le 700$

$x_{ik} \ge 0$, for $i=1,\ldots,3$; $k=1,\ldots 3$.

9 optimal variables in the optimal solution are

$x_{11} = 700,\ x_{13} = 800,$

And all others are zero.

This result points that the biggest global product will be reached if country 1 will become the only producer. This country is to deliver product 1 (700 units) and product 3 (800 units). Countries 2 and 3 should withdraw from doing anything and, naturally, resign form using a part of total resources. Product 2 should also disappear from factories, warehouses and shops.

Example 5. This will be the model (5), (6), (7) in case *P*. All resources can spread between all countries. We have 3 products *I=3*, 2 factors *J=2* and 3 countries *K=3*. The LP *3x3x2*problem has 6 constraints and 9 variables. All necessary data is to be found in table 11.

	Factor 1			Factor 2		
	Country 1	Country 2	Country 3	Country 1	Country 2	Country 3
Product 1	3	2	3	8	5	4
Product 2	4	2	5	10	5	8
Product 3	5	5	10	15	10	10
All resources	300	400	200	400	500	600

Tab. 11. Technological coefficients and resources.
Source: Example prepared by the author.

The problem (5), (6), (7) can be decomposed into *K* problems of smaller sizes (10), (11), (12). The data necessary for these problems are in tables 12, 13 and 14.

	Factor 1	Factor 2
Country 1	3	8
Country 2	2	5
Country 3	3	4
Resources at local level	300	400

Tab. 12. Technological coefficients and resources. Country 1.
Source: Example prepared by the author.

	Factor 1	Factor 2
Country 1	4	10
Country 2	2	5
Country 3	5	8
Resources at local level	200	400

Tab. 13. Technological coefficients and resources. Country 2.
Source: Example prepared by the author.

	Factor 1	Factor 2
Country 1	5	15
Country 2	5	10
Country 3	10	10
Resources at local level	500	600

Tab. 14. Technological coefficients and resources. Country 3.
Source: Example prepared by the author.

The starting LP problem is: maximize

$$x_{11} + x_{21} + x_{31} + x_{12} + x_{22} + x_{32} + x_{13} + x_{23} + x_{33}$$

subject to

$3\,x_{11} + 4\,x_{21} + 5\,x_{31} \le 300$

$2\,x_{12} + 2\,x_{22} + 5\,x_{32} \le 400$

$3\,x_{13} + 5\,x_{23} + 10\,x_{33} \le 200$

$8\,x_{11} + 10\,x_{21} + 15\,x_{31} \le 400$

$5\,x_{12} + 5\,x_{22} + 10\,x_{32} \le 500$

$4\,x_{13} + 8\,x_{23} + 10\,x_{33} \le 600$

$x_{ik} \ge 0$, for $i=1,2,3$; $k=1,2,3$.

Instead of the first problem we have 3 smaller problems, as follows

$$x_{11} + x_{21} + x_{31} \to \max$$

$3\,x_{11} + 4\,x_{21} + 5\,x_{31} \le 300$

$8\,x_{11} + 10\,x_{21} + 15\,x_{31} \le 400$

$x_{i1} \ge 0$, for $i=1,2,3$.

$$x_{12} + x_{22} + x_{32} \to \max$$

$2\,x_{12} + 2\,x_{22} + 5\,x_{32} \le 400$

$5\,x_{12} + 5\,x_{22} + 10\,x_{32} \le 500$

$x_{i2} \ge 0$, for $i=1,\dots,3$.

$$x_{13} + x_{23} + x_{33} \to \max$$

$3\,x_{13} + 5\,x_{23} + 10\,x_{33} \le 200$

$4\,x_{13} + 8\,x_{23} + 10\,x_{33} \le 600$

$x_{i3} \ge 0$, for $i=1,\dots,3$.

The optimal solution (positive values only) is

$$x_{13} = 100;\, x_{22} = \frac{400}{9} \approx 44{,}4;\, x_{23} = \frac{200}{9} \approx 22{,}2;\, x_{32} = 20;\, x_{33} = 40.$$

We came to the conclusion that country 1 will deliver only product 3, country 2 products 2 and 3, country 3 products 2 and 3. The products will be available in the world economy in the following amounts 100, 64,4 and 62,2.

What are the comparative advantages in our problem? This is a P case, where it is guaranteed that every country will produce something. And so it is as seen in table 15.

Countries	Comparative advantages
Country 1	Products 3
Country 2	Products 2 and 3
Country 3	Products 2 and 3

Tab. 15. Comparative advantages of the countries.
Source: Example prepared by the author.

Table 16 makes it possible to learn the absolute advantages of the countries concerned.

Countries \ Products	Product 1	Product 2	Product 3	Absolute advantages
Country 1	3+8	2+5	3+4	Products 1, 2, 3
Country 2	4+10	2+5	5+8	Products 2
Country 3	5+15	5+10	10+10	None

Tab. 16. Absolute advantages of the countries.
Source: Example prepared by the author.

Two countries 1 and 2 have absolute advantages in producing a good or a service 2. Country 3 has no absolute advantages, nevertheless it has comparative advantages in products 2 and 3.

Example 6. Let us be back to the example at the beginning of this text, we shall transform it as a LP problem. This will be a P case and model (5), (6), (7).

The world economy consists of 2 products $I=2$, a single factor $J=1$ and 2 countries $K=2$. Shirts are now product 1, computers are product 2, country A will be referred as country 1, country B as country 2. The global resources of labour, being the only one factor of production, is shared between 2 countries. The LP problem consists of 2 constraints and 4 variables. This starting problem is equivalent to 2 smaller LP problems with a single constraint and 2 variables. Tables in section 2 include all necessary data. Technological coefficients are inverses of labour productivity measures.

	Country A (country 1)		Country B (country 2)	
	Shirts	Computers	Shirts	Computers
Coefficients	1/6	1/4	1	1/2
Resources	4		12	

Tab. 17. Technological coefficients and resources.
Source: Example prepared by the author.

The LP problem is

$$x_{11} + x_{21} + x_{12} + x_{22} \to \max$$

$$\frac{1}{6}x_{11} + \frac{1}{4}x_{21} \le 4$$

$$x_{12} + \frac{1}{2}x_{22} \le 12$$

$x_{ik} \geq 0$, for $i=1,2$; $k=1,2$.

Only 2 variables in the optimal solution are positive

$$x_{11} = 24,\ x_{22} = 24.$$

Country 1 (country A) is manufacturing 24 units of product 1 (shirts), country 2 (country B) 24 units of product 2 (computers). This is the same result we have obtained before, to be seen in table 3.

Example 7. The example 1 and 6 will be reexamined. We shall assume that the total stock of labour is summed and available to every country. Still we have 2 products $I=2$, a single factor $J=1$ and 2 countries $K=2$. The LP problem consists of 4 variables and 1 constraint, before it had 2 constraints. The data are in table 18.

	Country A (country 1)	Country B (country 2)
Shirts (product 1)	1/6	1
Computers (product 2)	1/4	1/2
Resources worldwide	16	

Tab. 18. Technological coefficients and resources.
Source: Example prepared by the author.

The LP problem, the last one in this text, is

$$x_{11} + x_{21} + x_{12} + x_{22} \rightarrow \max$$

$$\frac{1}{6}x_{11} + \frac{1}{4}x_{21} + x_{12} + \frac{1}{2}x_{22} \le 16$$

$x_{ik} \geq 0$, for $i=1,2$; $k=1,2$.

Optimal positive values are

$$x_{11} = 96.$$

Under new circumstances one should just deliver 96 units of product 1 (shirts) in country 1 (country A). And this is all one should do to get the maximal world production.

7. Conclusions

It is possible to formulate the principle of comparative advantage using examples in a general case for any finite number of products m and countries n and resources J. One of the fundamental results of the international economics can have its generalized formulation which includes 2x2x1 cases.

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Ясіньські Лєшек. Узагальнення теорії відносних переваг. *Журнал Прикарпатського університету імені Василя Стефаника*, **4** (3-4) (2017), 21–33.

У підручниках з економіки стосовно принципу відносних переваг наводять приклади двох типів продукції та двох країн, випадок *2x2*. Ми запропонуємо підхід, що описує будь-яке кінцеве число продукції *m* та країни *n*, випадок *mxn*, де *m*> *2*, *n*> 2. З цією метою буде використано лінійне програмування.

Ключові слова: відносна перевага, абсолютна перевага, лінійне програмування, проміжні товари, технологічний коефіцієнт.