Journal of Vasyl Stefanyk Precarpathian National University
http://journals.pnu.edu.ua
Vol. 11, No. 1 (2024), 103-110

UDC 373.2.016:51]373.2.011.3-051
doi: 10.15330/jpnu.11.1.103-110

HEURISTIC METHOD OF G. POLYA IN THE OPINION OF THE EARLY-SCHOOL EDUCATION TEACHERS

EWELINA KAWIAK

Abstract. The proposed text discusses the issue of teaching mathematics at the first educational stage. Effective mathematics education is crucial for developing thinking skills and dealing with problem solving in everyday life. Pedagogical practice shows that problem-based methods are one of the most effective methods of teaching this subject. Taking into account the educational value of problem-based teaching methods, the heuristic method of G. Polya was particularly emphasized in the text. The method, created by a Hungarian scientist, is used to solve tasks. Solving problems, in turn, is one of the most important competencies developed during mathematics education. The examples of how the method has been used so far when working with younger pupils were demonstrated in the text below. The noticeable shortage of Polish literature regarding the application of this method when working with students of younger school age, encouraged the author to deeply reflect upon the current state of affairs. The article contains the results of the survey regarding the knowledge of G. Polya's method among early-school education teachers. The research was conducted in primary schools located in the Silesian Voivodeship. The study has attempted to define the opinions of the surveyed teachers with regard to the method and to establish what they consider to be the possibilities of using it in early-school education. The obtained research results show that despite many educational values, the described method is not very popular among the respondents. Therefore, it can be concluded that its capabilities are not used to a sufficient extent. These results encourage further reflection on possible ways of popularizing problem-based methods among professionally active teachers and among students preparing to work with students of early school age.

Keywords*:* heuristic method of G. Polya, mathematical education, problem solving, early-school education teacher.

1. INTRODUCTION

Teaching mathematics is one of the key elements of education in majority of educational systems. As a school subject, mathematics belongs to a group of obligatory subjects at all stages of school education, which makes mathematics one of the most significant contents being taught. According to a current National Curriculum, general education in a primary school aims to develop critical thinking skills and logical thinking among pupils as well as to accustom pupils to practice reasoning skills, argumentation and conclusion making (Budnyk, Nikolaesku, Solovei, et al., 2023). Effective teaching of mathematics should be conducted since early years because as the studies show that children's early mathematical experiences are essential for their further development of mathematical notions (Tudge & Doucet, 2004). The quality of mathematical classes in which the pupils participate has an impact on the level of their knowledge and competences. Educators of mathematics emphasize that the best results from the point of view of pedagogical theory and practice are achieved by problem-solving teaching of mathematics.

Contemporary theories regarding teaching-learning of this subject emphasize the role of problem solving. Letting pupils solve problems unassisted brings remarkable results during mathematical classes. Pupils' personal experiences play a significant role here, thanks to which, according to a constructivist and functional understanding of mathematical education, they have an opportunity to structure their knowledge. The ability to solve problems creatively should be one of the basic skills in which a student becomes equipped in the course of education (Kemple & Nissenberg, 2000). It is therefore legitimate to implement problem solving methods and heuristics during mathematics classes. The usage of heuristic methods while teaching-learning mathematics leads to the increase of students' competences in terms of unassisted and creative problem solving (Doulík et al., 2016). Accordingly, the teachers of early school education should include them among the methods they are already using and apply them regularly in mathematical classes.

2. THEORETICAL BACKGROUND

Among many methods of mathematical education which are based on problem solving, it is worth underlining the method which was devised by George Polya, a Hungarian mathematician living in 1887-1985. György (George) Polya is considered to be a creator of modern heuristics. This author is known in Poland mainly thanks to his two works. The first most well-known publication is "How to solve it?" (Polya, 1945), which was translated into Polish by Leszek Kubik in 1968. The other one is "Mathematical Discoveries" (Polya, 1962) translated by Andrzej Góralski in 1975.

The method by G. Polya belongs to the latest methods of classic strand of reflexive heuristics. Heuristic reasoning is a kind of reasoning which is not regarded as final and definite, but as provisional and merely probable, and its aim is to discover the solution to a given task or problem (Polya, 1945). The purpose of modern heuristics in G. Polya's approach is to understand the process of task solving. It focuses especially on understanding the mental operations which are most frequently used in this process. The method recommended by this Hungarian mathematician is of general nature, and task solving happens gradually, taking one step at a time, as follows: understanding the task, establishing the plan of action, execution of the plan, verification of the result and the reflection upon the solution, in other words, a look in hindsight (Polya, 1945; Góralski, 1980). It is worth emphasising that in the process of effective development of mathematical concepts, appreciation of childhood experiences plays a significant role (Semerádová, 2015). Students' independence in knowledge acquisition and thereby, in the process of task solving, is an essential element, underlined in G. Polya's method. The fundamental aim of teaching, according to G. Polya is to teach pupils how to think, whereas the essence of teaching mathematics is to make pupils discover as much as possible themselves in given conditions (Polya, 1962).

In Polish literature there are very few mentions of G. Polya's heuristic method's application when working with pupils at early school age. The person who made an attempt at implementing this method when working with pupils was Edmund Stucki. He demonstrated that this method has a positive impact on educational performance, developing mathematical thinking among pupils and it also helps to develop their interest in mathematics (Stucki, 1994). Validity of application of selected heuristic strategies of task solving, including G. Polya's method, when working with a younger student was also emphasised by Monika Wojnowska. According to her, the methods help students understand the contents of a mathematical task and help organize the information included in it (Wojnowska, 2007). It is worth mentioning that G. Polya's works and the premises of his method of problem solving were introduced to Polish readers by Andrzej Góralski- a Polish mathematician, theoretician and heurist, who died in 2020 (Góralski, 2013). Having observed a certain insufficiency of application of the above-mentioned method when working with pupils in first few school years, the author decided to analyse the opinions on that topic of the teachers of grades 1-3.

3. RESEARCH OBJECTIVE, METHODOLOGY AND DATA

The study which was conducted in the format of diagnostic survey, envolved 213 professionally active teachers of early school education, working in primary schools of Silesian Voivodship. The aim of the conducted survey was to obtain the answer to the question: "What are the opinions about the application of G. Polya's heuristic method of the surveyed teachers of the early school education?" The variables distinguished within the study were: location, the respondents' workplace, level of their education and teachers' work experience in a given school. Verification of the data obtained in the course of study was conducted by means of statistical conclusion with the use of rho-Spearman test and the "chi-square" test for independence. All tests were performed at the level of statistical significance α = 0,05.

4. RESULTS AND DISCUSSION

On the basis of the analysis of the obtained data, it was established that most of the surveyed teachers were not familiar with G. Polya's heuristic method. From among all the respondents, only 79 teachers admitted the knowledge of the method, which constitutes only 37% of the entire cohort. Having conducted statistical analyses it transpired that the singled-out variables do not have a significant impact on the familiarity with G. Polya's heuristic method among surveyed teachers.

Further analysis of the obtained results was based on the data gained from the questionnaires among the teachers who declared to be familiar with G. Polya's method. Consequently, the number of the surveyed teachers was limited here to 79 people, which makes up for 37% of all the teachers participating in the study.

The teachers familiar with G. Polya's method were asked to indicate its characteristic traits (table 1). It helped to determine in what way the respondents understand the proposed method.

Tab. 1

The responses describing G. Polya's method indicated by the surveyed teachers familiar with G. Polya's method

Response	Number	%
This method may be applied in early school education	68	86
It is a heuristic method	63	80
This method describes the subsequent stages of problem solving	53	67
It may be applied when solving any mathematical task	39	50
This method is not popular in a Polish school	26	33
This method is popular in a Polish school	25	32
This is an algorithmic method.	12	15
Total	286	363

Source: author's own research

Vast majority of the respondents (86%) claim that this method may be successfully used when working with pupils in early school education. A numerous group of respondents (80%) rightly qualified the method as the heuristic one. It is worth noticing that at the same time as many as 15% of surveyed teachers mistakenly described G. Polya's method as algorithmic method. For 67% of

respondents, the method describe the subsequent stages of problem-solving mathematical tasks. As a matter of fact, this is a characteristic feature of this method. Half of the surveyed teachers claims that with the use of G. Polya's heuristic method, every mathematical task can be solved. These opinions prove the universal character of this method. What is interesting, a comparable group of respondents is of a completely different opinion on the issue of popularity of the analysed method in Polish schools. One third of the respondents (33%) claim that G. Polya's method is popular in Polish school, whereas 32% insist that this is the opposite way round.

Nearly half of the surveyed teachers (40%) claim that they use G. Polya's method often, namely, at least once a week. Nearly one third of the respondents (32%) use it several times per month during their mathematical lessons. The performance of the above-mentioned teacher groups should be regarded as proper because regular application of the method when working with children increases the chances of realizing the assumed educational objectives. The remaining teachers indicated the usage of the method several times per semester or more seldom. Such rare application of the method raises some doubts. The effectiveness of teaching is closely connected with regularity and repetitiveness of the interactions between a teacher and his students.

In order to demonstrate the correlation between the school work experience of the surveyed teachers and the frequency with which they apply G. Polya's method when working with pupils in mathematical classes, the rho-Spearman correlation analysis was conducted. Table 2 presents the results of the test statistics.

Tab. 2

The value of rho-Spearman test for the correlation between the frequency of use of G.Polya's method by the surveyed teachers familiar with G. Polya's method and their school work experience

Variable	**Work experience**	
	rho-Spearman	**value p**
The frequency of use of G.Polya's method when working with pupils within mathematical education lessons	**-0,24**	**0,039**

Source: author's own research

It appeared that the shorter work experiences the surveyed teachers had, the more often they used G. Polya's method in their work with pupils during mathematical education classes. In that case, a value p = 0,039, and therefore $p < \alpha$ ($\alpha = 0,05$) was obtained. This result may signify that the less experienced teachers are more open-minded and willing to apply varied educational methods in their work with children. It may be conjectured that the teachers with longer work experience are accustomed to applying the tried and tested methods which they know well, therefore they diversify their teaching methods less willingly than the teachers who are beginning in this profession.

In order to verify whether the respondents of different educational level differed in terms of frequency of their application of G. Polya's method when working with children within mathematical education classes, the analysis based on the 'chi-square' test for independence was conducted. Table 3 presents the results for the indicated test statistics.

Tab. 3

The value of the 'chi-square' test for the dependence between the frequency of the application of G. Polya's method by the surveyed teachers and their level of education

Response	Education				Chi test result²	Value p
	Master's degree		Bachelor's degree			
	N	%	N	%		
At least once a week	30	39,00	1	33,33		
Several times per month	26	35,00	0	0,00		
Several times per semester	16	22,00	1	33,33	**9,47**	**0,034**
1-2 times per school year	3	3,00	0	0,00		
I do not use it	0	0,00	1	33,33		
No data	1	1,00	0	0		
Total	76	100	3	100		

Source: author's own research

As it was demonstrated, the teachers with master's degree applied G. Polya's method more frequently when working with their pupils in mathematical education classes than teachers with bachelor's degree – a value $p = 0,034$ was obtained, and therefore $p < \alpha$ ($\alpha = 0,05$). Such a result may imply that perhaps in the course of university education G. Polya's heuristic method is explained as late as during master's degree studies. Therefore, there is a need to increase university academics' awareness as to the necessity of inclusion of various methods of solving problem tasks, including G. Polya's heuristic method as early as in the bachelor's degree studies. This content should be taught within the module devoted to methodology of mathematical education. Making students acquainted with numerous educational advantages of G. Polya's method may cause that the students will apply its assumptions in their future teaching profession.

The teachers were asked to indicate in which grade they most often use G. Polya's method in their work with pupils. According to the obtained results, as many as 70% of teachers do not connect the application of use G. Polya's method with the grade in which they teach mathematics. They use the above-mentioned method successfully in all three grades of early school education. As many as 22 % of surveyed teachers to some extent differentiate it, by applying the method only in second and third grades of elementary school. Few teachers admitted to using this method rarely, only in grade II (1%) or only in grade II (5%). One teacher claims that despite knowing G. Polya's method, he does not use it in their work with pupils. A universal nature of G. Polya's method makes it attractive for early school education teachers. The elements of this method may be successfully introduced during mathematical classes as early as in first grade of elementary school. In subsequent years of school education, pupils should develop their efficiency in the usage of the heuristics described here.

According to majority of the surveyed teachers, the heuristic method of G. Polya positively influences pupils' competences. The respondents indicated which skills can be developed thanks to the application of this method in mathematics classes with younger pupils (Tab. 4).

Tab. 4

Opinions of the surveyed teachers familiar with G. Polya's method about the skills which can be developed in mathematics classes with the use of G. Polya's method

Response	Number of responses	%
Creative thinking skill	62	78
Independent thinking skill	60	76
Ability to make independent choices as to the problem-solving method	50	63
Perseverance in achieving one's goal	47	59
Team work ability	42	53
Critical thinking skill	28	35
Ability to accept failure	18	23

Source: author's own research

According to the biggest group of respondents (78%), application of G. Polya's method during mathematical education classes develops creative thinking and independent thinking ability (76%). Indeed, the study by E. Stucki, cited previously in the article, demonstrated the impact of G. Polya's method on pupils' thinking skills. In the opinion of 63% of respondents, application of G. Polya's method cultivates children's ability to make independent choices as to the problem-solving method and 59% respondents indicated that thanks to this method, children train their perseverance in achieving their goals and practice team work ability (53%). More than one third of the respondents believe that thanks to G. Polya's method, children's critical thinking improves. The surveyed teachers also admitted that application of G. Polya's method positively impacts children's ability to accept failure (23%), which is such an essential skill when we think of learning mathematics. The surveyed teachers observe positive effects of the application of G. Polya's method, both in reference to pupils as well as to their own work.

The quality analysis of the respondents' answers demonstrated that they notice numerous positive effects of the application of G. Polya's method in mathematical classes from the point of view of their profession. One of those is the possibility of observing the pupils during lesson work. According to 91 % of the respondents, the application of this method makes it possible to discern the mathematically gifted pupils. Significant 72% of teachers claim that the likelihood of diagnosing pupils with maths learning difficulties is high. A considerable group of teachers (76%) reckon that G. Polya's method enables pupils to make independent choices regarding preferred task solving methods. Thanks to this, it is possible to recognize pupils' preferred forms of lesson work. More than half of the respondents point out that the application of G. Polya's method allows teachers to evaluate pupils' engagement in the process of mathematical tasks solving, thereby giving grounds to formulate conclusions concerning pupils' attitudes towards learning of mathematics.

Respondents also determined the negative effects of application of G. Polya's method in mathematical classes. In their eyes, the negative aspects of using this method include loud behaviour of the pupils (59% responses) and no possibility to predict the final effects of their work (53%). These elements seem to be natural while conducting the lessons based on the premises of problem-based education, therefore, as such should not be perceived by teachers as negative. A part of the respondents (29%) observes that using heuristic methods, including the above mentioned one, does not guarantee that pupils will obtain a correct result while solving problem- based tasks. According to the respondents, it may affect the involvement of the pupils and lower their task motivation. Unpredictability of the lessons as a negative aspect was mentioned by 34% of the respondents. Organizational chaos during lessons as a negative aspect was problematic for 15% of respondents. A similar percentage of teachers (12%) claim that absolute lack of time control and no supervision over pupils' work may be seen as

another negative aspect of using G. Polya's method. What is interesting, the effects of using G. Polya's method, which the respondents assessed as negative, are in fact its indispensable elements, which in a way results from the specifics of using heuristic methods.

In literature, these elements are even indicated as the illuminating values of the methods. Negative assessment of the surveyed teachers is therefore puzzling and may signify teachers' lack of understanding of the significance of the application of this kind of methods as well as unwillingness to implement heuristics in their work. This is a worrying phenomenon which requires effective remedial measures.

5. CONCLUSION

Teachers who declared being acquainted with G. Polya's method had an opportunity to formulate their opinions on that topic, opting for either positive or negative aspects of its application in their work. As the study revealed, the respondents indicated numerous educational values of the described method, and thereby admitted to using it during the lessons. The teachers stated that the method positively influences pupils' cognitive sphere and raises their competence in terms of independent thinking and develops the skill to solve mathematical problems. Frequent application of G. Polya's method in mathematical classes with early school children is a rational activity. It should be remembered that teaching/learning with the use of heuristic methods brings pupils plenty of advantages such as the development of mathematical thinking and gaining the proficiency in problem solving. Such competences are an indispensable element of effective learning of mathematics and, in some way, they make educational success much more likely. The characteristics of G. Polya's method indicated as negative ones should stimulate reflection over the way in which it is depicted by its practitioners.

On the basis of the results of the questionnaire survey, it may be concluded that the knowledge of G. Polya's heuristic method among surveyed teachers is not satisfactory. Only one in three respondents declared being familiar with this method. Therefore, it is justified to state that the opportunities of applying G. Polya's method when working with young children are not taken advantage of to a satisfying extent. Such a situation may be caused by a small number of Polish methodical and scientific elaborations regarding this method. That is the reason why promoting G. Polya's method in academic environment as well as among teachers and students preparing for a teaching profession is legitimate. Activities such as those may contribute to the method's increase in popularity and to its more frequent application during classes with early-school children. When promoting optimal implementation of problem-solving methods, including G. Polya's method, in mathematics classes in grades 1-3, it is crucial, naturally, to remember about the quality education of the teachers. An essential element of that scheme is also to provide active teachers with an opportunity of professional development at the highest possible level. The realization of the above-mentioned demands will as a result contribute to the improvement of quality of mathematical education among early-school children.

REFERENCES

[1] Budnyk, O., Nikolaesku, I., Solovei, Y., Grebeniuk, O., Fomin, K., & Shynkarova, V. (2023). Paulo Freire's critical pedagogy and modern rural education. *Rev. Bras. Educ. Camp*., 8, e16480. http://dx.doi.org/10.20873/uft.rbec.e16480

[2] Doulík, P., Eisenmann, P., Přibyl, J., & Ńkoda, J. (2016) Unconventional Ways of Solving Problems in Mathematics Classes. *The New Educational Reviev*, *43, 1*. https://doi.org/10.15804/tner.2016.43.1.04

[3] Tudge, J. R. H., & Doucet, F. (2004). Early mathematical experiences: Observing young Black and White children's everyday activities. *Early Childhood Research Quarterly*, 19(1), 21–39. https://doi.org/10.1016/j.ecresq.2004.01.007

[4] Kemple, K. M., & Nissenberg, S. A. Nurturing Creativity in Early Childhood Education: Families Are Part of It. *Early Childhood Education Journal*, 28, 67–71 (2000). https://doi.org/10.1023/A:1009555805909

[5] Polya, G. (1945). *How to solve it?* Princeton: Princeton University Press.

[6] Polya, G. (1962). *Mathematical Discovery: On Understanding, Learning and Teaching Problem Solving*. *Vol. 1, 2.* New York - London: John Wiley & Sons, INC.
[7] Semerádová, S. (2015). *Didactical situations in building children`s ideas about matematical concepts in preschool education*. *Didactica Mathematicae*, *37*, 75-91.
[8] Stucki, E. (1992). *Heuristic method of G. Polya in early school mathematical education. Życie szkoły*, 10, 588-592. (in Polish)
[9] Wojnowska, M. (2007). *Between the message and discovery: creative techniques of solving mathematical tasks by children.* Oficyna Wydawnicza "Impuls". (in Polish)

Ewelina Kawiak, PhD, Institute of Pedagogy, Faculty of Social Science, University of Silesia in Katowice, Poland;
ORCID ID: 0000-0002-4719-7352
Address: Ewelina Kawiak, Grażyńskiego 53, 40-007 Katowice, Poland.
E-mail: ewelina.kawiak@us.edu.pl
Received: January 19, 2024; **revised:** January 31, 2024; **accepted:** February 27, 2024; **published:** March 19, 2024.

Кавяк Евеліна. Евристичний метод Г. Поля в оцінці педагогів дошкільної освіти. *Журнал Прикарпатського університету імені Василя Стефаника*, **11** (1) (2024), 103-110.

У пропонованому тексті представлено деякі питання викладання математики на першому освітньому етапі. Обґрунтовано, що ефективне навчання математики має вирішальне значення для розвитку навичок мислення та розв'язання проблем у повсякденному житті, адже, як засвідчує педагогічна практика, проблемні методи є одними з найефективніших у викладанні. Враховуючи їхню освітню цінність, у статті виокремлено евристичний метод угорського вченого Г. Поля, що полягає у розв'язуванні задач для формування компетентностей у процесі вивчення математики. Автором презентовано приклади використання цього методу в роботі з молодшими школярами. Помітний дефіцит польської літератури щодо застосування цього методу в роботі з учнями молодшого шкільного віку спонукав автора до глибокого осмислення окресленої проблеми. Стаття містить результати опитування щодо знання методу Г. Поля, яке проведено серед учителів початкових класів, розташованих у Сілезькому воєводстві. У дослідженні здійснено спробу визначити думку опитаних учителів щодо цього методу та встановити, якими вони бачать можливості його використання в дошкільній освіті. Отримані результати дослідження показують, що, незважаючи на багато освітніх цінностей, даний метод не користується великою популярністю серед респондентів. Тому акцентовано на його можливостях для практичного застосування. Ці результати спонукають до подальших роздумів щодо деяких шляхів популяризації проблемних методів серед професійно активних педагогів і студентів, які готуються до роботи з учнями молодшого шкільного віку.

Ключові слова: евристичний метод Г. Поля, математична освіта, розв'язування задач, педагог дошкільної освіти.