ФІЗИКА І ХІМІЯ ТВЕРДОГО ТІЛА
Т. 20, № 1 (2019) С.5-12
DOI: 10.15330/pcss.20.1.5-12

PHYSICS AND CHEMISTRY OF SOLID STATE
V. 20, № 1 (2019) P.5-12

UDK 539.2

SSN 1729-4428

I.A. Konstantinovich[1,2], A.V. Konstantinovich[1]

Radiation Spectrum of Sequence of Electrons Moving in Spiral in Medium

[1]Chernivtsi National University, Kotsybynsky Str.,2, Chernivtsi, 58012, Ukraine,
E-mail: dj_kneo@ukr.net; aconst@ukr.net
[2]Institute of Thermoelectricity NAS and MES of Ukraine, Nauky Str., 1, Chernivtsi, 58029, Ukraine

Integral expressions for spectral-angular and spectral distributions of the radiation power for the sequence of electrons moving in magnetic fields in isotropic transparent medium are investigated using the improved Lorentz`s self-interaction method. Special attention is given to the research of the fine structurre of the spectral distribution of the synchrotron-Cherenkov radiation of one, two, three and four point electrons moving along the spiral in medium. The effects of coherent radiation of harmonics and oscillations in spectrum of the synchrotron-Cherenkov radiation of two, three and four point electrons are established and investigated using the direct numerical method for calculation the function of spectral distributions of the radiation power.

Key words: synchrotron-Cherenkov radiation, sequence of electrons, fine structure of spectrum, effects of coherence, oscillations in radiation spectrum.

Work arrived to the editor 12.10.2018.; accepted for printing15.03.2019.

Introduction

Larmor [1-4] for the first time established that a single charged point particle, which moves with acceleration in vacuum, always radiates electromagnetic waves. The Larmor formula for the power of radiation of point charged particles, which was obtained for the nonrelativistic case, was generalized by the Lienard [5] and Heaviside [6] to the relativistic case.

In 1907, Schott [7-9] for the first time strictly within the framework of classical electrodynamics investigated the radiation spectrum of electrons that move in a circle in vacuum. Later G.A. Schott developed classical theory of the radiation of charges moving in the circle and used it for the investigations of a model of atom [9]. His attempt to explain the radiation of the atom on the basis of classical physics was not successful. Due to these reasons, the work of Schott over the course of 40 years has only become an area of academic interest and has practically been forgotten. After 40 years, the Schott formula has been applied to the study of the radiation spectrum of charged particles moving on a macroscopic scale (synchrotron radiation). The main properties of synchrotron radiation of charged particles that move in a magnetic field in vacuum are analyzed in a review of [10] and monographs [11-14].

The Schott formula [10] being only of academic interest for so long period of time is also related to the fact that Schott only in 1933 [15] established the conditions under which the distributions of charged particles moving with acceleration and performing periodic motion, do not emit electromagnetic waves. Interest in this class of distributions of charged particles and their fields is also due to the possibility of their application to classical, stable models of elementary particles, atoms, and in other cases [16-20].

The radiation spectrum of a sequence of non-interacted charged particles that move along a spiral in vacuum is investigated in [21, 22]. Superhigh-power short-wave coherent synchrotron radiation by a sequence of charged particles bunches was studied in [23-25].

The classical theory of radiation emitted by charged particles moving with superluminal velocities were traced back to Heaviside [26], Des Condres [27], and Sommerfeld [28-31]. The classical theory of the Cherenkov phenomenon in a dispersive medium was first formulated by Frank and Tamm in 1937 [32].

The peculiarity in the radiation of charges and multipoles uniformly moving in a medium is analyzed in monographs [33-36].

The generalized Cherenkov-like effects based on four fundamental interactions have been investigated and classified in [37].

Current results from Cherenkov radiation near the Cherenkov barrier [36, 38-40] and from anomalous Cherenkov rings [41, 42] stimulated new theoretical studies in this area [43-44].

Tsytovich [45], for the first time, examined the case of oscillations in the radiation spectrum of a relativistic charged particle, which moves in a circle, in a constant magnetic field in a medium with dispersion.

Above the Cherenkov barrier, for electrons that move in a spiral, the appearance of oscillations [46-50] and hopping changes [51-53] of the function of spectral distribution of radiation power was established.

The aim of this work is to obtain by the improved Lorentz self-interaction method the basic formulae for the spectral-angular and spectral distributions of the time-average power of the radiation of a sequence (system) moving along an arbitrary given trajectory in a transparent isotropic medium. Using the numerical method of direct numerical calculation of the spectral distribution function of the radiation power of electrons, the fine structure of the radiation spectrum of a sequence of electrons moving along a spiral in a magnetic field in a transparent isotropic medium was studied. Considerable attention is paid to the study of oscillations and coherent radiation near the Cherenkov barrier.

I. Time-Averaged Radiation Power of Charged Particles Moving in a Transparent Medium

According to [46, 47, 54, 55] the time-averaged radiation power $\overline{P}^{\,rad}$ of charged particles moving in a transparent isotropic medium is determined by the relationship:

$$\overline{P}^{rad}=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}P^{rad}(t)dt=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt\times$$
$$\times\left\{\int_{\tau}\left(\vec{j}(\vec{r},t)\frac{1}{c}\frac{\partial\vec{A}^{Dir}(\vec{r},t)}{\partial t}-\rho(\vec{r},t)\frac{\partial\Phi^{Dir}(\vec{r},t)}{\partial t}\right)d\vec{r}\right\},\qquad(1)$$

Here, $\vec{j}(\vec{r},t)$ is the current density and $\rho(\vec{r},t)$ is the charge density. The integration is performed over some volume τ . According to the hypothesis of Dirac [55, 56], the scalar $\Phi^{Dir}(\vec{r},t)$ and vector $\vec{A}^{Dir}(\vec{r},t)$ potentials are defined as a half-difference of the retarded and advanced potentials

Retarded and advanced scalar $\Phi^{ret,\,adv}(\vec{r},t)$ and vector $\vec{A}^{ret,\,adv}(\vec{r},t)$ potentials of charged particles moving in a medium, taking into account the frequency dispersion of the dielectric $\varepsilon(\omega)$ and magnetic $\mu(\omega)$ permeabilities, are determined by the relations [54, 55]:

$$\Phi^{ret,adv}(\vec{r},t)=\frac{1}{4\pi^{3}}\int_{-\infty}^{\infty}d\vec{r}'\int_{-\infty}^{\infty}dt'\int_{-\infty}^{\infty}d\vec{k}\int_{-\infty}^{\infty}d\omega\frac{\rho(\vec{r}',t')}{\varepsilon(\omega)}\times$$
$$\times\frac{\exp\{i\vec{k}(\vec{r}-\vec{r}')-i\omega(t-t')\}}{k^{2}-\frac{n^{2}(\omega)}{c^{2}}(\omega\pm i\alpha)^{2}},\qquad(2)$$

$$\vec{A}^{ret,adv}(\vec{r},t)=\frac{1}{4\pi^{3}}\int_{-\infty}^{\infty}d\vec{r}'\int_{-\infty}^{\infty}dt'\int_{-\infty}^{\infty}d\vec{k}\int_{-\infty}^{\infty}d\omega\mu(\omega)\frac{\vec{j}(\vec{r}',t')}{c}\times$$
$$\times\frac{\exp\{i\vec{k}(\vec{r}-\vec{r}')-i\omega(t-t')\}}{k^{2}-\frac{n^{2}(\omega)}{c^{2}}(\omega\pm i\alpha)^{2}},\qquad(3)$$

Here, α is a positive infinitesimal, which turns into zero after integration, refraction index $n(\omega)=\sqrt{\varepsilon(\omega)\mu(\omega)}$.

The instantaneous radiation power $P^{rad}(t)$, which is expressed in terms of the spectral-angular distribution of the radiation power $W_1(t,\omega,\theta,\varphi)$ of charged particles, taking into account relations (1), (2), (3), takes the form:

$$P^{rad}(t)=\int_{0}^{\infty}d\omega\int_{0}^{2\pi}d\varphi\int_{0}^{\pi}\sin\theta d\theta W_1(t,\omega,\theta,\varphi),\qquad(4)$$

$$W_1(t,\omega,\theta,\varphi)=\frac{1}{4\pi^{2}c^{3}}\int_{-\infty}^{\infty}d\vec{r}\int_{-\infty}^{\infty}d\vec{r}'\int_{-\infty}^{\infty}dt'\omega^{2}\mu(\omega)n(\omega)\times$$
$$\times\cos\left\{\frac{n(\omega)}{c}\omega\sin\theta\left[\cos\varphi\,(x-x')+\sin\varphi\,(y-y')\right]\right\}\times$$
$$\times\cos\left[\frac{n(\omega)}{c}\omega\cos\theta\,(z-z')\right]\cos\omega(t-t')\times$$
$$\times\left[\vec{j}(\vec{r},t)\vec{j}(\vec{r}',t')-\frac{c^{2}}{n^{2}(\omega)}\rho(\vec{r},t)\rho(\vec{r}',t')\right],\qquad(5)$$

The instantaneous radiation power $P^{rad}(t)$, which is expressed in terms of the spectral-angular distribution of the radiation power $W_2(t,\omega,\theta)$, can be obtained from (4), (5) using the relation for the Bessel function of the integer index (see p. 416 in [57]):

$$\int_{0}^{2\pi}d\varphi\,\cos\left[\frac{n(\omega)}{c}\omega(\sin\theta\,\cos\varphi\,(x-x')+\sin\theta\,\sin\varphi\,(y-y'))\right]=$$
$$=2\pi J_0\left(\frac{n(\omega)}{c}\omega\sin\theta\sqrt{(x-x')^{2}+(y-y')^{2}}\right),\qquad(6)$$

where $J_0(x)$ is the Bessel function of zero index..

Then we find:

$$P^{rad}(t)=\int_{0}^{\infty}d\omega\int_{0}^{\pi}\sin\theta d\theta W_2(t,\omega,\theta),\qquad(7)$$

$$W_2(t,\omega,\theta)=\frac{1}{2\pi c^{3}}\int_{-\infty}^{\infty}d\vec{r}\int_{-\infty}^{\infty}d\vec{r}'\int_{-\infty}^{\infty}dt'\omega^{2}\mu(\omega)n(\omega)\times$$
$$\times J_0\left(\frac{n(\omega)}{c}\omega\sin\theta\sqrt{(x-x')^{2}+(y-y')^{2}}\right)\times$$
$$\times\cos\left[\frac{n(\omega)}{c}\omega\cos\theta\,(z-z')\right]\cos\omega(t-t')\times$$
$$\times\left[\vec{j}(\vec{r},t)\vec{j}(\vec{r}',t')-\frac{c^{2}}{n^{2}(\omega)}\rho(\vec{r},t)\rho(\vec{r}',t')\right],\qquad(8)$$

The instantaneous radiation power, which is expressed in terms of the spectral distribution of the radiation power, can be obtained from (7), (8) using the relation for the Bessel functions of an integer index (see p. 757 in 57]):

$$\int_0^{\frac{p}{2}} \sin q\, dq J_0(a \sin q)\cos(b \cos q) = \frac{\sin\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}, \qquad (9)$$

$$P^{rad}(t) = \int_0^{\infty} dw W_3(t,w), \qquad (10)$$

$$W_3(t,w) = \frac{1}{pc^2}\int_{-\infty}^{\infty} d\vec{r} \int_{-\infty}^{\infty} d\vec{r}\,' \int_{-\infty}^{\infty} dt' w m(w) \frac{\sin\left[\frac{n(w)w}{c}|\vec{r}-\vec{r}\,'|\right]}{|\vec{r}-\vec{r}\,'|}\times$$
$$\times \cos w(t-t')\left\{\vec{j}(\vec{r},t)\vec{j}(\vec{r}\,',t') - \frac{c^2}{n^2(w)} r(\vec{r},t) r(\vec{r}\,',t')\right\}, \qquad (11)$$

Relation (11) for the spectral distribution of the instantaneous radiation power $W_3(t,\omega)$, is obtained and investigated by Schwinger et al. [58], using the sources theory.

II. Spectral and spectral-angular distributions of the time-average radiation power of sequence of electrons moving along a spiral in a medium

The current density $\vec{j}(\vec{r},t)$ and charge density $r(\vec{r},t)$ of N non-interacting charged point particles are defined by relationships:

$$\vec{j}(\vec{r},t) = \sum_{l=1}^{N} \vec{V}_l(t) r_l(\vec{r},t), \quad r(\vec{r},t) = \sum_{l=1}^{N} r_l(\vec{r},t),$$
$$r_l(\vec{r},t) = e d\left(\vec{r}-\vec{r}_l(t)\right), \qquad (12)$$

where $\vec{r}_l(t)$ and $\vec{V}_l(t)$ are the motion law and the velocity of the l^{th} particle, respectively, $d(x)$ is the Dirac delta-function.

We study the case of the sequence of electrons moving one by one along a spiral in a transparent medium. The law of motion and the velocity of the l^{th} electron in magnetic field are given by the expressions:

$$\vec{r}_l(t) = r_0\cos[w_0(t+\Delta t_l)]\vec{i} + r_0\sin[w_0(t+\Delta t_l)]\vec{j} + V_{\parallel}(t+\Delta t_l)\vec{k},$$
$$\vec{V}_l(t) = \frac{d\vec{r}_l(t)}{dt}, \qquad (13)$$

Here, $r_0 = V_{\perp} w_0^{-1}$, $w_0 = c^2 e B^{ext}/\tilde{E}$, $\tilde{E} = c\sqrt{p^2+m_0^2c^2}$, the magnetic induction vector $\vec{B}^{ext}\,||0Z$, $V_{\perp}$ and $V_{\parallel}$ are the components of the velocity, $\vec{p}$ and $\tilde{E}$ are the momentum and energy of the electron, e and m_0 are its charge and rest mass, respectively, c is velocity of light in vacuum.

We will get the time-averaged radiation power of a sequence of electrons N by substituting (12), (13) into (1), (10), (11). Then we find [54, 55]:

$$\bar{P}^{rad} = \int_0^{\infty} W(w)dw, \qquad (14)$$

$$W(w) = \frac{2e^2}{pc^2}\int_0^{\infty} dx w m(w) S_N(w) \frac{\sin\{n(w) w c^{-1} h(x)\}}{h(x)}\cos(wx)\times$$
$$\times\left[V_{\perp}^2\cos(w_0 x) + V_{\parallel}^2 - \frac{c^2}{n^2|w|}\right], \qquad (15)$$

where $\eta(x) = \sqrt{V_{\parallel}^2 x^2 + 4\frac{V_{\perp}^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}$, $W(w)$ is the function of spectral distribution of the time-averaged radiation power, $m(w)$ is the magnetic permeability, $n(\omega)$ is the refraction index, ω is cyclic frequency, c is velocity of light in vacuum. The coherence factor $S_N(w)$ is determined by the expression

$$S_N(w) = \sum_{l,j=1}^{N}\cos\{w(\Delta t_l - \Delta t_j)\}, \qquad (16)$$

Here, Δt_l is the time shift of the l^{th} electron.

In the case of two electrons the coherence factor $S_2(w)$ is defined as:

$$S_2(w) = 2 + 2\cos(w\Delta t_{12}), \qquad (17)$$

Here, Δt_{12} is the time shift between the first and second electrons.

The coherence factor $S_3(w)$ of three electrons takes the form

$$S_3(w) = 3 + 2\cos(w\Delta t_{12}) + 2\cos(w\Delta t_{23}) +$$
$$+ 2\cos\{w(\Delta t_{12}+\Delta t_{23})\}, \qquad (18)$$

Here, Δt_{23} is the time shift between the second and third electrons.

The coherence factor $S_4(\omega)$ of four electrons is defined as

$$S_4(w) = 4 + 2\cos(w\Delta t_{12}) + 2\cos(w\Delta t_{23}) +$$
$$+ 2\cos(w\Delta t_{34}) + 2\cos\{w(\Delta t_{12}+\Delta t_{23})\} +$$
$$+ 2\cos\{w(\Delta t_{23}+\Delta t_{34})\} + 2\cos\{w(\Delta t_{12}+\Delta t_{23}+\Delta t_{34})\}, \qquad (19)$$

where Δt_{34} is the time shift between the third and fourth electrons.

After some transformations of relationships (14) and (15) the contribution of separate harmonics to the time-averaged radiation power can be expressed as:

$$\bar{P}^{rad} = \frac{e^2}{c^3}\sum_{m=-\infty}^{\infty}\int_0^{\infty} dw m(w) n(w) w^2 S_3(w)\int_0^{p}\sin q\, dq\times$$
$$\times d\left\{w\left(1-\frac{n(w)}{c}V_{\parallel}\cos q\right) - m w_0\right\}\times$$
$$\times\left\{V_{\perp}^2\left[\frac{m^2}{q^2}J_m^2(q) + J_m'^2(q)\right] + \left(V_{\parallel}^2 - \frac{c^2}{n^2(w)}\right)J_m^2(q)\right\}, \qquad (20)$$

where $q = \frac{n(w)}{c}\frac{w}{w_0}V_{\perp}\sin q$, $J_m(q)$ and $J_m'(q)$ are the Bessel function with integer index and its derivative, respectively.

Each harmonic is a set of the frequencies, which are the solutions of the equations

$$w\left(1-\frac{n(w)}{c}V_{\|}\cos q\right)-mw_0=0\,, \qquad (21)$$

The coherence factor of a single electron is defined as $S_1(w)=1$.

III. Oscillations in the spectrum of synchrotron-Cherenkov radiation of a sequence of electrons moving along a spiral in a medium

The functions of spectral distribution $W(w)$ of the synchrotron-Cherenkov radiation power of one, two, three and four electrons moving along a spiral in a medium are calculated according to (14), (15) for $B^{ext}=1\,Gs,$ $m=1$, $n=1.3$, $V_{\perp med}>c/n$, $V_{\perp med}=0.26\times10^{11}$ *cm/s,* $V_{\| med}=0.15\times10^{10}$ *cm/s,* $w_{0j}=0.8112\times10^{8}$ *rad/s,* $r_{0j}=2984\,cm$ (j=1, 2, ..., 12) (Figs 1-9).

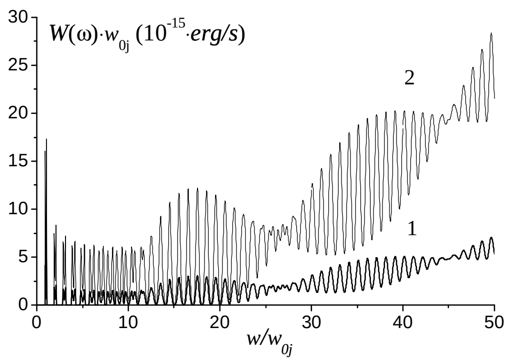

Fig. 1. Spectral distribution of the synchrotron-Cherenkov radiation power at low harmonics for $B^{ext}=1\text{Gs}$, $\mu=1$, $n=1.3$, $V_{\perp med}=0.26\times10^{11}$ *cm/s,* $V_{\| med}$ = 015 × 10 ^{10} *cm/s.* Curve 1 is calculated for the case of one electron with power $P^{\text{int}}_{med1}=0.11201\times10^{-12}$ *erg/s,* curve 2 is calculated for two electrons at time shift $\Delta t^2_{12}=0.0001\times p\,/w_{02}$ with radiation power $P^{\text{int}}_{med2}=3.994\times P^{\text{int}}_{med1}=0.44738\times10^{-12}$ erg/s.

The spectral distribution of synchrotron-Cherenkov radiation of one, two, three, and four electrons at low harmonics at $V_{\perp med}>c/n$, $n=1.3$, $V_{\perp med}=0.26\times10^{11}$ cm/s, $V_{\| med}=0.15\cdot10^{10}$ cm/s (Fig. 1-9) has the character of discrete bands. With further increase in harmonics number, the function of spectral distribution of synchrotron-Cherenkov radiation power of electrons takes on a near-periodical character due to the overlapping of the bands of neighboring harmonics and the contribution of other harmonics (Figs. 1-9). According to relation (21), the expansion of discrete harmonics into bands is due to the Doppler effect.

For small time shifts between the electrons for the system of two, three, and four electrons in the frequency range of $0-50w_{0j}$ we have found the existence of the coherent radiation $S_N(\omega)=N^2$, so far as the dimension of this system is smaller in comparison to the radiation wavelength (Figs.1-3). The sequence moving along a spiral radiates as a charged particle with a charge Ne and rest mass Nm_0, i.e. by a factor N^2 more than a single electron.

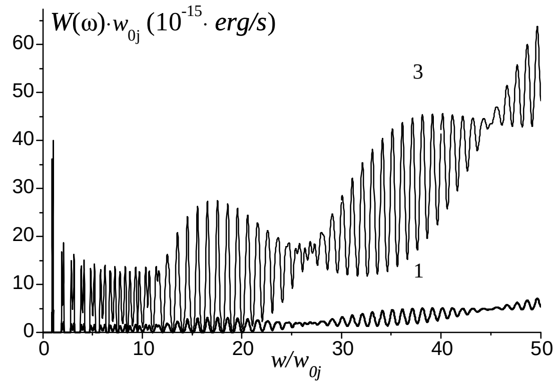

Fig. 2. Spectral distribution of the synchrotron-Cherenkov radiation power at low harmonics for $B^{ext}=1\text{Gs}$, $\mu=1$, $n=1.3$, $V_{\perp med}=0.26\times10^{11}$ *cm/s,* $V_{\| med}=0.15\times10^{10}$ *cm/s.* Curve 3 is calculated for three electrons at time shifts $\Delta t^3_{12}=\Delta t^3_{23}=0.0001\times p\,/w_{03}$ with power $P^{\text{int}}_{med3}=8.987\times P^{\text{int}}_{med1}=0.10066\times10^{-11}$ erg/s.

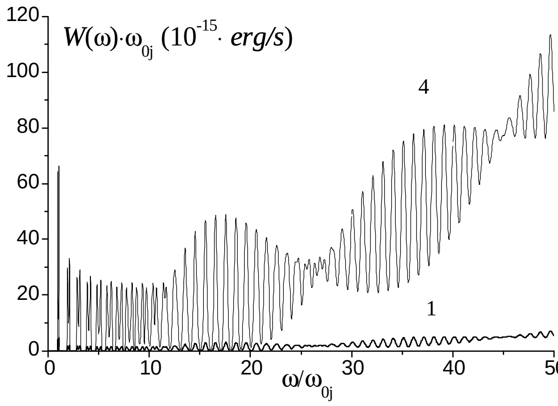

Fig. 3. Spectral distribution of the synchrotron-Cherenkov radiation power at low harmonics for $B^{ext}=1\text{Gs}$, $\mu=1$, $n=1.3$, $V_{\perp med}=0.26\times10^{11}$ *cm/s,* $V_{\| med}=0.15\times10^{10}$ *cm/s.* Curve 4 is calculated for four electrons at time shifts $\Delta t^4_{12}=\Delta t^4_{23}=\Delta t^4_{23}=0.0001\times p\,/w_{04}$ with power $P^{\text{int}}_{med4}=15.97\times P^{\text{int}}_{med1}=0.17892\times10^{-11}$ erg/s.

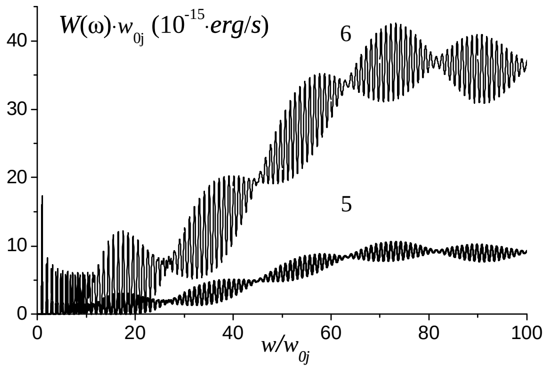

Fig. 4. Spectral distribution of the synchrotron-Cherenkov radiation power at low and middle harmonics for $B^{ext} = 1\,\mathrm{Gs}$, $\mu = 1$, $n = 1.3$, $V_{\perp med} = 0.26\times10^{11}$ *cm/s*, $V_{\parallel med}$ = 015 × 10 ^{10} *cm/s*. Curve 5 is calculated for the case of one electron with power $P^{\mathrm{int}}_{med5} = 0{,}54275\times10^{-12}$ *erg/s*, curve 6 is calculated for two electrons at time shift $\Delta t^{6}_{12} = 0.0001\times p\ /\ w_{06}$ with radiation power $P^{\mathrm{int}}_{med6} = 3{,}992\times P^{\mathrm{int}}_{med5} = 0.21668\times10^{-11}$ erg/s.

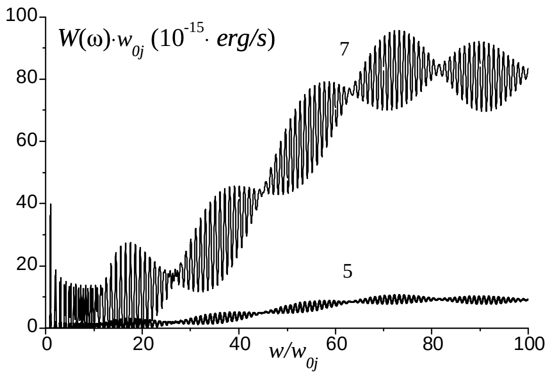

Fig. 5. Spectral distribution of the synchrotron-Cherenkov radiation power at low and middle harmonics for $B^{ext} = 1\,\mathrm{Gs}$, $\mu = 1$, $n = 1.3$, $V_{\perp med} = 0.26\times10^{11}$ *cm/s*, $V_{\parallel med} = 0.15\times10^{10}$ *cm/s*. Curve 7 is calculated for three electrons at time shifts $\Delta t^{7}_{12} = \Delta t^{7}_{23} = 0{,}0001\times\pi/\omega_{07}$ with power $P^{\mathrm{int}}_{med7} = 8{,}981\times P^{\mathrm{int}}_{med5} = 0.48744\times10^{-11}$ erg/s.

In the radiation spectrum of the studied system of electrons moving along a spiral at $V_{\perp med} > c/n$, oscillations of the function of spectral distribution of the power of synchrotron-Cherenkov radiation are observed (Figs. 4–9). At high harmonics at $V_{\perp med} = 0.26\times10^{11}$ *cm/s*, $V_{\parallel med} = 0.15\times10^{10}$ *cm/s*, the overlap of neighboring harmonics does not practically lead to near-periodical variations of the function of spectral distribution of the power of synchrotron-Cherenkov radiation of electrons, but only oscillations of this function are observed (Figs. 7–9). With decreasing the longitudinal component of velocity, the near-periodical variations of the spectral distribution of the synchrotron-Cherenkov radiation become more essential. The obtained results are in good agreement to those obtained in [50].

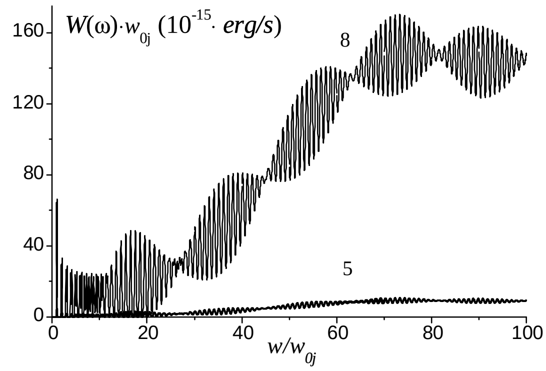

Fig. 6. **Sp**ectral distribution of the synchrotron-Cherenkov radiation power at low and middle harmonics for $B^{ext} = 1\,\mathrm{Gs}$, $\mu = 1$, $n = 1.3$, $V_{\perp med} = 0.26\times10^{11}$ *cm/s*, $V_{\parallel med} = 0.15\times10^{10}$ *cm/s*. Curve 8 is calculated for four electrons at time shifts $\Delta t^{8}_{12} = \Delta t^{8}_{23} = \Delta t^{8}_{23} = 0{,}0001\times p\ /\ w_{08}$ with power $P^{\mathrm{int}}_{med8} = 15{,}961\times P^{\mathrm{int}}_{med5} = 0.86631\times10^{-11}$ erg/s.

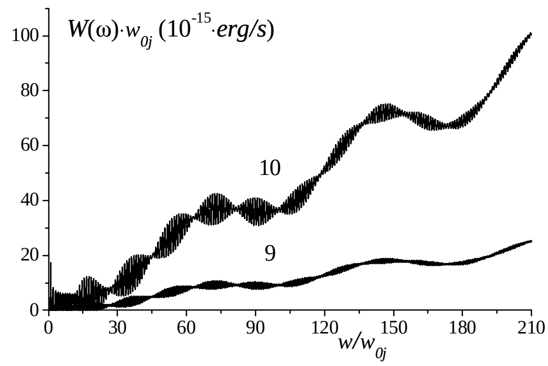

Fig. 7. Oscillations and near-periodical variations in synchrotron-Cherenkov radiation spectrum at low, middle, and high harmonics for $B^{ext} = 1\,Gs$, $m = 1$, $n = 1{,}3$, $V_{\perp med} = 0{,}26\times10^{11}$ *cm/s*, $V_{\parallel med} = 0{,}15\times10^{10}$ *cm/s*. Curve 9 is calculated for the case of one electron with power $P^{\mathrm{int}}_{med9} = 0{,}23923\times10^{-11}$ *erg/s*, curve 10 is calculated for two electrons at time shift $\Delta t^{10}_{12} = 0{,}0001\times p\ /\ w_{10}$ with power $P^{\mathrm{int}}_{med10} = 3{,}991\times P^{\mathrm{int}}_{med9} = 0.95484\times10^{-11}$ *erg/s*.

The oscillations of the function of spectral distribution of synchrotron-Cherenkov radiation power of one, two, three, and four electrons moving along a spiral in a medium at $V_{\perp med} > c/n$ is determined by the contribution of the Bessel functions [59] (Fig. 7-9). The numerical method of direct integration of the function of spectral distribution of radiation power of one, two, three and four electrons moving along a spiral in a medium allowed us to determine the fine structure of the radiation spectrum of these electrons

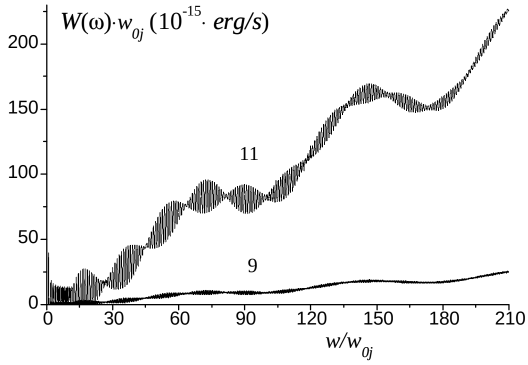

Fig. 8. Oscillations and near-periodical variations in synchrotron-Cherenkov radiation spectrum at low, middle, and high harmonics for $B^{ext} = 1\,\text{Gs}$, $m = 1$, $n = 1.3$, $V_{\perp med} = 0.26 \times 10^{11}$ cm/s, $V_{\| med} = 0.15 \times 10^{10}$ cm/s. Curve 11 is calculated for three electrons at time shifts $\Delta t_{12}^{11} = \Delta t_{23}^{11} = 0{,}0001 \times p\ / w_{011}$ with power $P_{med11}^{\text{int}} = 8{,}974 \times P_{med9}^{\text{int}} = 0.21468 \times 10^{-10}$ erg/s.

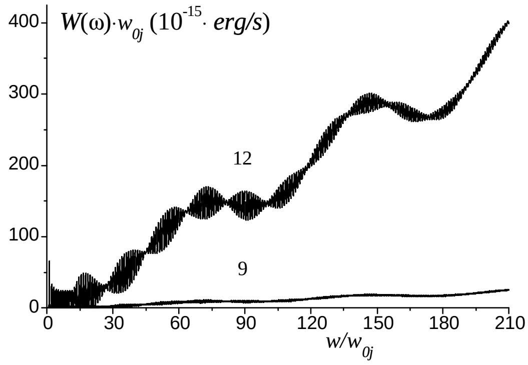

Fig. 9. Oscillations and near-periodical variations in synchrotron-Cherenkov radiation spectrum at low, middle, and high harmonics for $B^{ext} = 1\,Gs$, $m = 1$, $n = 1.3$, $V_{\perp med} = 0.26 \times 10^{11}$ *cm/s*, $V_{\| med} = 0.15 \times 10^{10}$ *cm/s*. Curve 12 is calculated for four electrons at time shifts $\Delta t_{12}^{12} = \Delta t_{23}^{12} = \Delta t_{23}^{12} = 0{,}0001 \times p\ / w_{012}$ with power $P_{med12}^{\text{int}} = 15{,}934 \times P_{med9}^{\text{int}} = 0.38119 \times 10^{-10}$ *erg/s*.

These studies confirm the fact that the synchrotron-Cherenkov radiation of one, two, three, and four electrons is an unified process [53].

Conclusions

1. For a small longitudinal velocity component at low harmonics, the radiation bands of electrons, moving along a spiral in a medium, are discrete.
2. The influence of the Doppler effect determines the boundaries of the bands of individual harmonics in the spectra of synchrotron-Cherenkov radiation of one, two, three and four electrons, moving along a spiral in a medium.
3. For small time shifts between the electrons for the sequence of two, three, and four electrons in the frequency range of $0 - 50w_{0j}$ we have found the existence of the coherent radiation $S_N(\omega) = N^2$ so far as the dimension of this system is smaller in comparison to the radiation wavelength. The sequence moving along a spiral radiates as a charged particle with a charge Ne and rest mass Nm_0, i.e. by a factor N^2 more than a single electron.
4. The oscillations of the function of spectral distribution of synchrotron-Cherenkov radiation power of one, two, three, and four electrons moving along a spiral in a medium at $V_{\perp med} > c/n$ is determined by the contribution of the Bessel functions.
5. It is confirmed that the synchrotron-Cherenkov radiation of one, two, three and four electrons is an unified process. The influence of the Doppler effect on the structure of the spectral distribution of the power of the radiation of electrons becomes significant near the Cherenkov barrier.

Konstantinovich I.A. – Ph.D of physical and mathematical sciences, docent
Konstantinovich A.V. – doctor of physical and mathematical sciences, docent.

[1] J. Larmor, Philosophical Transactions of the Royal Society 190, 205 (1897).
[2] J. Larmor, Philos. Mag. 44, 503 (1897).
[3] J. Larmor, Aether and Matter (Cambridge, University Press, 1900),
[4] J. Jackson, Classical Electrodynamics (Mir, Moscow, 1965).
[5] A.M. Liénard, L'Eclairage électique 16, 5 (1898).
[6] O. Heaviside, Nature 67, 6 (1902).
[7] G.A. Schott, Annalen der Physik 24(14), 635 (1907).
[8] G.A. Schott, Philos. Mag. 13, 189 (1907).
[9] G.A. Schott, Electromagnetic Radiation and the Mechanical Reactions arising from It (Cambridge: University Press, 1912),
[10] I.M. Ternov, UFN 165(4), 429 (1995). (doi.org/10.3367/UFNr.0165.199504c.0429).
[11] A.A. Sokolov, I.M. Ternov, Relativistic electron (Science, Moscow, 1974).
[12] V.A. Bordovitsyn, I.M. Ternov, Synchrotron Radiation Theory and Its Development in Memory of I.M. Ternov, (Singappore: Word Scientific, 1999).

[13] H. Wiedemann, Synchrotron radiation (Berlin and Heidelberg: Springer-Verlag, 2003).
[14] A. Hofmann, The Physics of Synchrotron Radiation (Cambridge: University Press, 2007).
[15] G.A. Schott, Phil. Mag. 15(7), 752 (1933).
[16] D. Bohm, M. Weinstein, Phys. Rev., 74, 1789 (1948).
[17] G.H. Goedecke, Phys. Rev. B, 135 (1), 281 (1964).
[18] P. Pearle, “Classical Electron Models”, in Electromagnetism: Paths to Research, ed. D. Teplitz (Plenum Press, New York, 1982).
[19] H.A. Haus, American Journal of Physics 54(12), 1126 (1986).
[20] R.L. Mills, The Grand Unified Theory of Classical Quantum Mechanics. V.1, V.2, V.3 (BlackLight Pover, 2010).
[21] A.V. Konstantinovich, I.A. Konstantinovich, Romanian Reports in Physics 58(2), 101 (2006).
[22] A.V. Konstantinovich, I.A. Konstantinovich, Journal of Optoelectronics and Advanced Materials 8 (6), 2143 (2006).
[23] Y. Pinhasi, A. Gover, Nucl. Instr. Meth. Phys. Res. A. 358(1-3), 86 (1995). (doi.org/10.1016/0168-9002(94)01417-5).
[24] J. Neumann, R. Fiorito, H. Freund et al, Proceedings of the FEL Conference (Trieste, Italy, 2004). P.86.
[25] V.A. Bordovitsyn, V.G. Bulenok, T.O. Pozdeeva, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms 227(1), 143 (2005). (doi.org/10.1016/j.nimb.2004.05.012).
[26] O. Heaviside, Phil. Mag. 27(5-th series), 324 (1889).
[27] Th. Des Condres, Arch. Néer 5, 652 (1900).
[28] A. Sommerfeld, Nachr. Königl. Ges. Wiss. Göttingen 99 (1904).
[29] A. Sommerfeld, Nachr. Königl. Ges. Wiss. Göttingen, 363 (1904).
[30] A. Sommerfeld, Nachr. Königl. Ges. Wiss. Göttingen, 201 (1905)
[31] A. Sommerfeld, Kon. Ned. Akad, Weten. Amsterdam 7, 346 (1905).
[32] I.E. Tamm, I.M. Frank, CCA Academy Reports 14(3), 107 (1937)
[33] J. Jelly, Cherenkov radiation (Moscow: IL, 1960).
[34] V.P. Zrelov, Vavilov-Cherenkov Radiation (Atomizdat, Moscow, 1968).
[35] I.M. Frank, Vavilov-Cherenkov Radiation. Questions of theory (Science, Moscow, 1988).
[36] G.N. Afanasiev, Vavilov-Cherenkov and Synchrotron Radiation: Foundations and Applications (Kluwer Academic Publishers, Dordrecht-Boston-London, 2004).
[37] D.B. Ion, W. Stocker, Phys. Rev. C, 52(6), 3332 (1995). (doi.org/10.1103/PhysRevC.52.3332).
[38] V.G. Kartavenko, G.N. Afanasiev, W. Greiner, Physica B: Condensed Matter. 271(1-4), 192 (1999).
[39] T.E. Stevens, J.K. Wahlstrand, R. Kuhl, R. Cherenkov, Science 291(5504), 627 (2001).(DOI: 10.1126/science.291.5504.627).
[40] M. Čiljak, J. Ruzicka, A.S. Vodopianov, et al., Nuclear Instruments and Methods in Physics Research Section A, 498(1-3), 126 (2003).(doi.org/10.1016/S0168-9002(02)01920-4).
[41] A.S. Vodopianov, V.P. Zrelov, A.A. Tyapkin, Particles and Nuclei, Letters 2(99), 35 (2000).
[42] A.S. Vodopianov, Y.I. Ivanshin, V.I. Lobanov et al., Nuclear Instruments and Methods in Physics Research Section B, 20 (1), 266 (2003).
[43] D.B. Ion, E.K. Sarkisyan, Rom. J. Phys., 49 (1-2), 25 (2004).
[44] D.B. Ion, E.K. Sarkisyan, Rom. J. Phys., 49(7-8), 671 (2004).
[45] V.N. Tsytovich, Bulletin of Moscow University. Physics (11), 27 (1951).
[46] A.V. Konstantinovich, I.A. Konstantinovich, Astroparticles Physics 30 (3), 142 (2008). (doi.org/10.1016/j.astropartphys.2008.07.006).
[47] A.V. Konstantinovich, I.A. Konstantinovich, Problems of Atomic Science and Technology. Series: Nuclear Physics Investigations, (5), 67 (2011).
[48] A.V. Konstantinovich, I.A. Konstantinovich, Rom. J. Phys. 56(1-2), 4552 (2011).
[49] A.V. Konstantinovich, Spectra of radiation of relativistic and nonrelativistic electrons and their sequence in a vacuum and a transparent environment, Thesis for a doctor's degree in physics and mathematics, Chernivtsi national university, 330 p. (2012).
[50] A.V. Konstantinovich, I.A. Konstantinovich, Rom. Rep. Phys. 66(2), 307 (2014).
[51] A.V. Konstantinovich, S.V. Melnychuk, I.A. Konstantinovich, Journal of Optoelectronics and Advanced Materials 5(5), 1423 (2003).
[52] A.V. Konstantinovich, I.A. Konstantinovich, Bulletin of the Kharkiv National University named after V.N. Karazin Series "Radiophysics and Electronics", 983 (19), 38 (2011).
[53] A.V. Konstantinovich, I.A. Konstantinovich, Rom. J. Phys. 57 (9-10), 1356 (2012).
[54] A.V. Konstantinovich, S.V. Melnichuk, I.M. Rarenko, I.A. Konstantinovich, V.P. Fizzy, Journal of Physical Research 4 (1), 48 (2000).
[55] A.V. Konstantinovich, I.A. Konstantinovich, Physics and Chemistry of Solid State 8 (2), 240 (2007).
[56] P.A.M. Dirac, Proc. Roy Soc. A, 167 (1), 148 (1938).
[57] I.S. Gradshtein, I.M. Ryzhik, Tables of Integrals, Sums, Rows, and Works (Nauka, Moscow, 1971).

[58] J. Schwinger, Tsai Wu-yang, T. Erber, Ann. of Phys. 96 (2), 303 (1976); 281 (1-2), 1019 (2000).
[59] E. Yanke, F. Emde, F. Lesch, Special Functions (Science, Moscow, 1964).

І.А. Константинович[1,2], А.В. Константинович[1]

Спектр випромінювання послідовності електронів, що рухаються вздовж гвинтової лінії в середовищі

[1]Чернівецький національний університет, вул. Коцюбинського 2, Чернівці, 58012, Україна,
e-mail: dj_kneo@ukr.net; aconst@ukr.net
[2]Інститут термоелектрики Національної академії наук і Міністерства освіти і науки України,
вул. Науки 1, Чернівці, 58029, Україна

Удосконаленим методом сили самодії Лоренца досліджено інтегральні вирази спектрально-кутового й спектрального розподілів потужності випромінювання послідовності електронів, що рухаються в магнітному полі в прозорому ізотропному середовищі. Особливу увагу приділено дослідженню тонкої структури спектрального розподілу потужності синхротронно-черенковського випромінювання одного, двох, трьох та чотирьох точкових електронів, що рухаються вздовж гвинтової лінії в середовищі. Розробленим методом прямого числового розрахунку функції спектрального розподілу потужності випромінювання встановлено й досліджено ефекти когерентного випромінювання гармонік та осциляції в спектрах синхротронно-черенковського випромінювання послідовності двох, трьох та чотирьох точкових електронів.

Ключові слова: синхротронно-черенковське випромінювання, послідовність електронів, тонка структура спектра, когерентні ефекти, осциляції в спектрі випромінювання.