PHYSICS AND CHEMISTRY OF SOLID STATE
V. 25, No. 4 (2024) pp. 704-717
Section: Physics

DOI: 10.15330/pcss.25.4.704-717

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ФІЗИКА І ХІМІЯ ТВЕРДОГО ТІЛА
Т. 25, № 4 (2024) С. 704-717
Фізико-математичні науки

PACS: 78.67.Sc; 81.05.ue

ISSN 1729-4428 (Print)
ISSN 2309-8589 (Online)

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Free vibration analysis for polyester/graphene nanocomposites multilayer functionally graded plates

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In the current work, an FGM nanocomposite made of multi-layers of graphene nanoparticles and a polyester-based matrix was constructed using the molds with a hand lay-up technique to attain an accurate shape and reduce defects in the final product. The desired models have four, six, and 11 layers and different volume fractions of nanoparticles (0.5, 1, 2, 3, 4, and 5%). This study conducted various experimental tests to analyze the free vibration characteristics of functionally graded composite sandwich rectangular structures with simply supported boundary conditions to evaluate the significance of hole number and cutout location in fundamental frequencies. For holes, three types are used (2, 4, and 6) holes with a 10 mm diameter, while for cutouts, three geometrical designs are used (circular, rectangular, and triangular) with various aspect ratios (r = 1, 1.5, 2, and 2.5).

A numerical study was carried out to validate the experimental solution, employing modal analysis and finite element analysis (FEA) using ABAQUS software tools. From the findings, the experimental findings and numerical calculations exhibit a satisfactory level of concurrence, displaying a maximum discrepancy of 9.5%. The results show that the fundamental frequency decreases by increasing the cutouts' aspect ratio (r = a/b). There is minimal variation between r =1 and r = 1.5, but a noticeable decrease is observed at an aspect ratio of r = 2.5. This difference is primarily influenced by the type of material gradient and the number of holes, specifically for a given thickness of the functionally graded (FG) plates.

Keywords: FGM, Nanocomposite structure, Geometrical Properties, Natural frequency, ABAQUS.

Received 20 February 2024; Accepted 08 October 2024.

Introduction

In various industries, composite structures have proven helpful over unreinforced materials because of their advantages. Due to the benefits such arrangements offer, such as lightweight and high specific strength, these structures are gaining attention in many applications, including aerospace, automobiles, energy, construction, and biomaterial science [1-3].

Functionally graded materials (FGMs) are a sophisticated category of composite materials recognized for their distinctive design characteristics that allow them to carry out diverse functions and serve various applications [4-7]. Burlayenko and Sadowski (2019) examined the modal and stability response examination of simply supported FGSP with variable proportions of the (Al_2O_3) constituent using 3D finite elements models. The natural frequencies, mode shapes, displacements, and stresses within ABAQUS were determined utilizing the FEM for the FGSP. The numerical findings demonstrate a remarkable correlation with analytical solutions, showcasing the reliable performance of a developed 3-D graded finite element. It is crucial to highlight that, to achieve convergence, the mesh size should be kept below 0.5 mm [8]. A study by Wang et al. (2021) [9] examined the capability of FG corrugated structures using FSDT. The researchers employed the differential quadrature finite element method to establish a comprehensive dynamic model for FGCPS. The proposed approach was validated using the ABAQUS finite element software to ensure convergence, stability, and accuracy. The model's shape was found to align well with the results obtained from ABAQUS for different boundary conditions. The research findings indicated that the functional grading (FG) parameters reliably forecasted the performance of desired models.

Leana S. et al. (2021) used experimental and finite element methods to study dynamic analyses of LCP. This study examined multiple attributes, such as the model layering method, geometrical arrangement, and BCs. Using the Lab. view software platform, the results obtained by rig arrangement are changed from the time domain, and the natural frequencies are obtained and compared with another numerical approach [10].

Njim et al. (2021) [11] presented a comprehensive mathematical model for describing the free vibration problem in an FG rectangular sandwich plate with a simply support system. The sandwich plate's core is made from a porous material. The classical plate theory determines the unconstrained vibration parameters of defective media. The sandwich of FGM ratings involves porosity, power-law exponent, core metal kind, and various aspect ratios. The FEA and modal analysis were performed numerically in ANSYS-2020-R2 to verify the analytical solution. Analytical solutions and numerical results agree within 8%.

Duc and Minh's study [12] examined functionally graded carbon nanotube-reinforced composite (FG CNTRC) plates with cracks for free vibration properties. Models with CNT are evenly arranged throughout the thickness direction of the model, and the equations of motion for cracked plates are formulated using the finite element method and 3rd-order shear deformation theory (TSDT). The study found that the number of carbon nanotubes (CNTs) per plate, geometrical dimensions, crack characteristics, and boundary conditions all impact the dynamic response of the (FG CNTRC) plates. When the crack length ratios (c/L), plate length ratios (L/H), and thickness ratios (H/H) decrease, the stiffness of the plate also decreases, resulting in a reduction in the frequency of free vibration. Moreover, it is noticed that the order of the boundary conditions also affects the vibration frequency.

FG CNTRC plates can help design and optimize composite structures for various applications because this computational simulation provides insights into the factors influencing free vibration behavior.

An analysis of free vibration and bending of ribbed plates with a hole was presented by XC. He et al. (2022) [13]. In addition to Hamilton's Principle, the entire transformation method is used to enforce the boundary conditions governing the static and dynamic behaviors of the plates. Comparing the proposed method with ABAQUS and other widely used commercial software, the results obtained by the proposed method are comparable. The presented method is highly effective and accurate in analyzing ribbed plates' static and dynamic problems with holes. Overall, this study contributes to the advancement of numerical methods for analyzing the mechanical behavior of ribbed plates with a hole and provides a practical and efficient tool for engineers and researchers in this field.

Varun Gopalakrishnan et al. (2022) [14], The study's main focus was to examine the integrity behavior of the C-resin layered model, having different geometrical configurations. The circular plates were made of a carbon-epoxy composite material commonly used in various engineering applications. Non-perforated composite plates were subjected to free motion using ANSYS to study the effect of height, radius, and fiber arrangement on dynamic properties. It also examined how cutout dimensions relate and how the plate's frequencies can help design carbon-epoxy composite plates with specific vibrational characteristics. Engineers and designers can utilize this information to optimize the performance and behavior of such plates in various applications where control of vibrations is crucial, such as in developing lightweight structures with high stiffness and reduced resonance effects.

The complementary effects of different materials on sandwich plates were investigated by Jing Zhao et al. (2022) [15]. This study proposes a unified modeling method to analyze the behavior of magnetorheological elastomer sandwich plates with functionally graded graphene-reinforced layers. This method combines the modified Fourier series and Rayleigh-Ritz method to generate a mathematical formulation for accurately predicting the plate's response. The results obtained from the analysis reveal that the presence of graphene plates (GPLs) causes a noticeable rise in the resonance frequencies, implying that GPLs enhance the stiffness of the plate and improve its vibrational properties.

Quansheng Zang et al. (2023) [16] developed an analytical plate formulation based on a 3D elastic approach to examine the static bending and dynamic behavior of (FGM) plates. A power-law scheme was used to represent the mathematical model. FGM plates were also evaluated for static and free vibration responses based on their volume fraction index and geometrical characteristics. The results show that FG parameters significantly influence the mechanical properties and free vibration characteristics.

Junsheng Zhu et al. (2023) [17] investigated the steadiness of the skew SP using FSDT. There are three layers in the sandwich plate: a metal foam core, a pure metal face, and a plastic coating. Pores are distributed in the core through different types of functionally graded patterns. An eigenvalue problem can be derived using the Ritz method, in which Chebyshev polynomials can represent shape functions. According to the results, plate aspect ratio, host-to-face thickness ratio, skew angle, and pores' pattern and size influence free vibration characteristics.

Jagesh Kumar Prusty et al. (2023) [18] examined SP's supper imposed loading involvement with a viscoelastic core and cutouts through experimentation and numerical analysis. The assessment of model response depends on how the cutouts are positioned, sized, and shaped. Electronic impulse hammers carry out the modal test. Later, the finite element technique is employed for the numerical analysis. ANN models are then created using experimental and numerical data to forecast SP deformation. A sizable dataset is used to train and test the ANN models, and different statistical measures are used to assess their accuracy. The excellent agreement between the results from the two approaches supports the accuracy of the numerical simulation approach.

A modified plate theory was used to analyze a graphene-surrounded composite piezoelectric layer studied by Ma et al. (2023) [19]. By incorporating the transverse shear stress field and Hamilton's principle, the natural frequencies of piezoelectric laminated plates can be predicted. The accuracy of this new theory is assessed by comparing its results with exact solutions and outcomes obtained from other methods. The proposed approach shows improved accuracy in predicting natural frequencies compared to existing higher-order models. Additionally, a parametric study is conducted to investigate the vibration responses of the composite plates with graphene reinforcement, considering various graphene and piezoelectric plate parameters. Examining the mechanical properties of advanced materials is crucial because they are frequently used in engineering. According to the literature review described above, most researchers are interested in investigating FGM plates' static and free vibration. However, the impact of the geometrical characteristics of functionally graded nanocomposite structures based on polyester has not received much research. The literature concludes that Simulating various functionally graded polymer composite properties has been the subject of extensive research. The experimental work done on these composites is also insufficient. There have been extensive investigations into polymer-based functionally graded materials, but the author does not know of any synthesized nanographene-based functionally graded materials.

This research aims to understand how the FGM structure with different geometrical characteristics behaves under various loading scenarios using numerical and experimental methods. The aim of the study can therefore be summarized as follows:

Models are fabricated using two methods: One uses nanographene with epoxy volume fractions in varying grades using a hand lay-up technique, and another uses neat polyester with different constituents of nanoparticles.

This study compares the tensile strength, hardness, and three-point bend of different FGMs by performing mechanical tests on graphene resin composites and comparing the effects of changing reinforced material constituents.

Investigate experimentally the free vibration behavior of sandwich plates that incorporate functionally graded material with and without holes and cutouts.

Examining natural frequency variation and deformation based on geometrical properties and comparing experimental and finite element models using various ABAQUS tools.

I. Materials And Methods

In this study, (TOPAZE 1110 TP), an unsaturated polyester resin manufactured by Industrial Chemicals & Resins Ltd. (ICR), widely used in composite structure fabricating, was incorporated as a liquid matrix because of its low viscosity, good physical properties, and mechanical stability. The basic properties of (TOPAZE 1110 TP) are listed in Table (1), while Graphene nanoparticle reinforcements are obtained from Skyspring Nanomaterials, Inc., with characteristics shown in Table (2).

Table 1.

Technical properties of (TOPAZE 1110 TP) polyester resin (from the datasheet of supplier)

Compressive strength (MPa)	**Flexural strength (MPa)**	**Tensile strength (MPa)**	**Density (g/cm^3)**
50	80	50	1.15

Table 2.

Properties of Graphene nanopowder (from the Skyspring nanomaterials supplier)

Properties	**Values**
Appearance	Black powder
Purity (%)	99.5
Average particle size (nm)	15
Morphology	Platelet
Thickness (nm)	6-8
Surface Area (m^2/g)	120-150

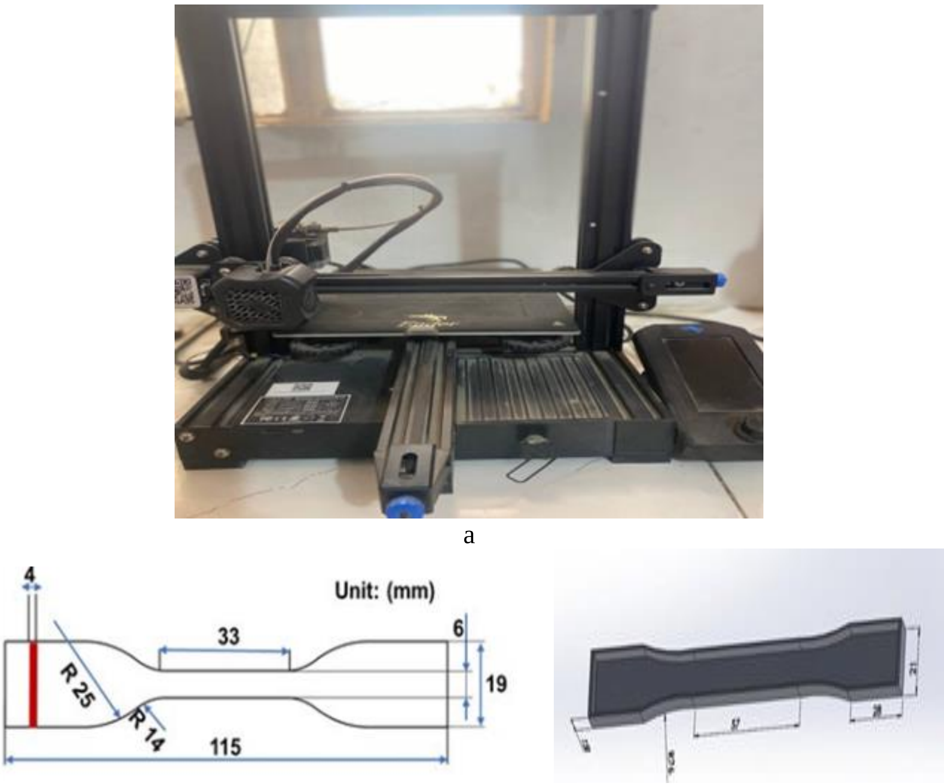
A solid 3D model is generated using the molds with inserts method, using data from previously discussed volume or mass fraction ratio formulas. Each object layer is constructed by sequentially layering materials, with a slice of the finished product being formed at the bottom of each layer. A mold with inserts enables a more complex design when compared with subtractive manufacturing, which eliminates material through other manufacturing methods. Additionally, this approach has the benefit of using less material than conventional manufacturing techniques to produce detailed designs. In this study, two methods are used to create the molds.

Using a CNC (computer numerical control) machine, the first die type was created, then ground to produce composite samples with various dimensions following international standard specimens. The models for the free vibration test were prepared using wooden molds with a 6 mm thickness.

The second kind of dies was a rectangular mold with dimensions of (110 x 1104) mm^3, and tensile and three-point bending molds were created using a 3D printer. In this work, polylactic acid (PLA) polymer filament with a diameter of 1.75 mm manufactured by China was the raw material for creating tensile and flexural bending molds using an Ender - 3D printer machine, as shown in Figure (1 a).

The 3D printed specimens were designed, exported as STL files, and imported into a 3D printing machine using SolidWorks software. Figure (1b) displays the main dimensions of the samples. The printing settings used for each sample are displayed in Table (3), with a bed temperature of 60C. Wooden molds for the fabrication of free-vibration rectangular plates and beams are shown in Figure (2).

Composite nanomaterials can be made from polymers, metals, ceramics, and fillers such as graphene, nanotubes, and clays. These fillers enhance material properties, such as their mechanical, thermal, and electrical properties. Many diverse industrial fields utilize polymeric nanocomposites, including energy, electronics,



a

b c

Fig. 1. (a) A mold fabricated by 3D printing used for (b) a 3-point bending sample (ASTM D790), (c) a Tensile test sample according to ASTM D 638 (all dimensions in mm).

biomedicine, etc.

Table 3.
Printing parameters used for all mold sample manufacturing

Nozzle dia. (mm)	Layer Thick. (mm)	Infilling density	Printing T (°C)	Printing speed (mm/sec)
0.40	0.28	100%	200	50

Fig. 2. A wooden mold for free vibration composite FGMs sample (300 x 300 x 10 mm).

A liquid polyester matrix was used in this study; graphene nanoparticles were utilized as nanofillers. Hand-layup techniques were used to synthesize nanocomposites. In order to produce a consistent mixture, a minimal amount of a suitable solvent (thinner) was poured over the resin before the reinforcement addition. To create equal situations between the neat polyester samples and other models, the polyester resin was dissolved in a thinner solvent in sufficient quantities for preparation. A mechanical stirrer was used to blend the mixture for 10 minutes, and then it was put in a vacuum vessel for the same period. It is essential that the solvent completely evaporates under a vacuum produced by a vacuum pump. Once the mixture was mixed, it was poured into a desired mold and cured at room temperature for two days.

However, the desired reinforcement amounts for preparing isotropic graphene/nanoparticles are (0.5, 1, 1.5, 2, and 4 %) V_f graphene nanoparticles dissolved in an adequate amount of the specified solvent. The obtained mixture was sonicated for ten minutes at a frequency of seventy-five percent after being blended to make it uniform for ten minutes with a magnetic stirrer. A similar method as described earlier was used to add polyester to such a mixture. The air bubbles were removed by vacuum pumping for 10 minutes after being mechanically mixed for a few minutes. The homogeneous mixture was then placed in the wooden mold and cured for two days. This process flow diagram for creating samples is shown in Figure (3).

Preparation of FGNCM

Table 4 overviews the functionally graded nanocomposite materials (FGNCM) samples. The process of creating functionally graded nanocomposites involved two distinct phases. The first phase involved preparing a mixture of nanographene and polyester matrix through effective spreading and mixing techniques discussed in the mentioned procedure above (Figure 4). This step was explicitly designed for the production of isotropic nanocomposites. In the second phase, the liquid mixture was sequentially poured into wooden molds, layer by layer, following a predetermined graded structure. Layers that were 1.2 mm thick were released into the desired molds.

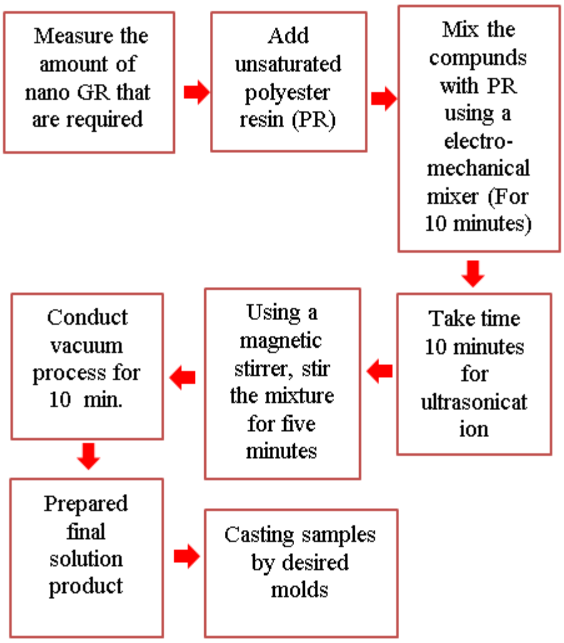


Fig. 3. The flow chart used in the experimental work to prepare a mixture of FGM sample fabricating.

Table 4.

Functionally graded composite samples with different weight fractions

a. FGM Type 1 (4-ply)

No. of layers	Graphene (wt %)	Polyester (wt %)
1	0	100
2	6.666	93.34
3	13.34	86.66
4	20	80

b. FGM Type 2 (6-ply)

No. of layers	Graphene (wt %)	Polyester (wt %)
1	0	100
2	4	96
3	8	92
4	12	88
5	16	84
6	20	80

c. FGM Type 3 (11-ply)

No. of layers	Graphene (wt %)	Polyester (wt %)
1	0	100
2	2	98
3	4	96
4	6	94
5	8	92
6	10	90
7	12	88
8	14	86
9	16	84
10	18	82
11	20	80

For FGM Sample 1, the number of layers is four, and the weight fraction details for each layer are indicated in Table 4 a. The Thickness of each layer introduced into a mold is (1.2 mm). As for FGM Sample 2, the composite was cast with six layers according to the weight fraction in the same Table (4 b) with a total height of (10 mm). The same technique was used for fabricating FGM sample 3 but with a total number of layers 11.

The homogenous composite samples are fabricated for comparative study. Different volume fractions of graphene reinforcement are used (0.5, 1, 1.5, 3, and 4) % as filler in combination with polyester (matrix) to manufacture a homogenous composite. Figures 4 and 5 show the fabrication of FG samples using the hand-up method with wooden molds.

Fig. 5. The hand lay-up technique used for fabricating composite FGM samples.

II. The Experimental Work

Tensile Test Experiments

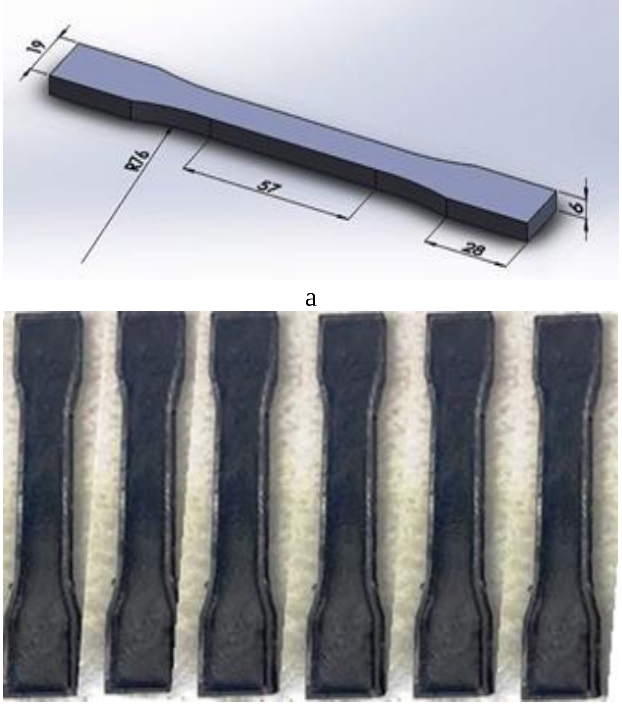
It is common practice to conduct tensile test experiments on nanocomposite and functionally graded nanocomposite samples to assess their mechanical characteristics and performance under tensile loading. In order to ensure that quality standards are upheld, it is critical to determine whether they comply with the specifications listed in the material specifications. The experiment involves applying controlled tension to manufactured specimens until they deform or fracture.

Figure (6a) presents the photo and specimen dimensions and illustrates the specimen's geometry. The 3D-printed polymer samples (Figure 6b) were created following ASTM standard D638 [20]. Figure 7 depicts the tests conducted on a (50 kN) UTM microcomputer-controlled electronic universal testing machine (Tinius Olsen H50KT machine).

The FG samples' tensile strength was restricted to improve the results' accuracy. All tensile tests were conducted at 1 mm/min cross-head speed. The results were generated as forces in (N) versus specimen deformation in (mm) on a digital computer as the load was applied. An average reading was taken from five samples for each test in order to ensure accuracy.

Free Vibration Test of FG Sandwich Plates

Sandwich plate models from FGPM oscillated without external excitation or applied forces during the free vibration test. This test aimed to look into the sandwich plate FGMs' natural frequencies, mode shapes, and damping characteristics. We learned much about the plates' dynamic behavior and structural features by examining how they responded to the free vibrations. The sandwich plates were tested by measuring their displacements, accelerations, and other relevant parameters to identify their natural frequencies and vibration modes. These results help us comprehend the performance and design factors for sandwich plates made of FGMs in various engineering applications. Several factors, including FG constituents, gradient index, number of layers, and volume fraction index, were considered when testing samples for free vibration [21-23].



a

b

Fig. 6. (a) Dimensions of the tensile sample (mm) following ASTM standard D638 [20]. (b) Manufactured polymer tensile test specimens.

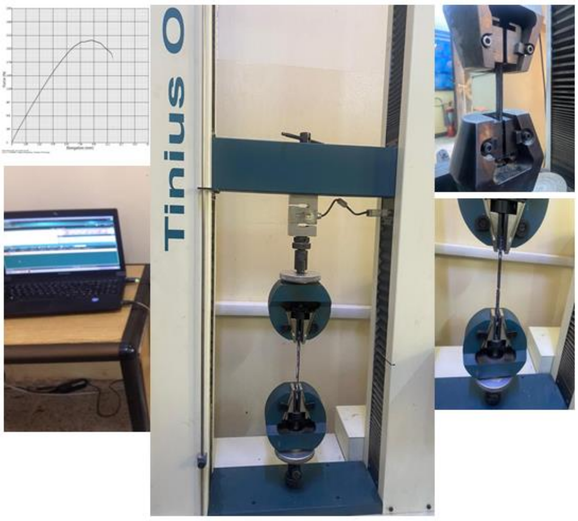
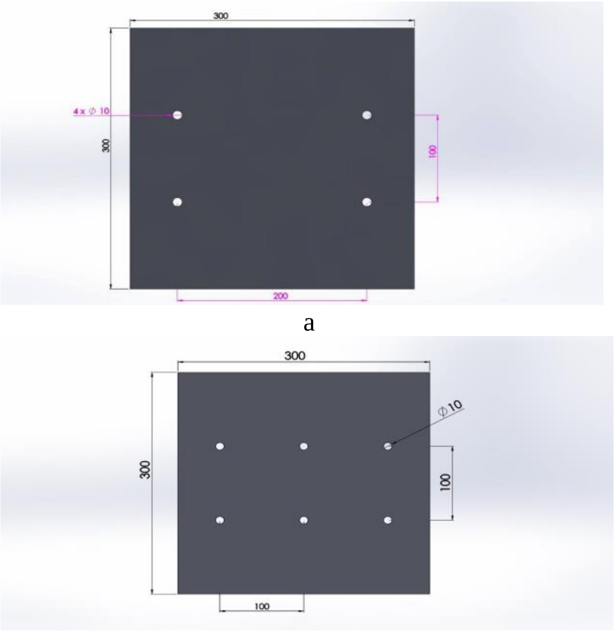


Fig. 7. Tensile Test setup.

Wooden molds are used to fabricate samples of free vibration tests. The sandwich plate with and without holes is made using wooden molds and manual additive of the raw materials. Figure 8 shows the geometry of FG sandwich plate samples with holes, while Figure 9 illustrates the geometry with the triangular cutout sample.

The fundamental natural frequency of rectangular sandwich plates made of functionally graded nanocomposite materials can be ascertained by performing free vibration tests. Modal analysis techniques were used on a test bench designed specifically for vibration analysis to determine the characteristics of a functionally graded rectangular sandwich plate with holes and cutouts. FGM sandwich plates (300*300*10) mm with varying holes and cutouts, geometrical properties, and layers were used in this test. The plates have a gradient of two, four, and six holes with (dia.=10) mm and four types of cutouts (rectangular, square, triangular, and elliptic).



a

b

Fig.8. Schematic geometry for free vibration sample containing (a) 4 holes, (b) 6 holes.

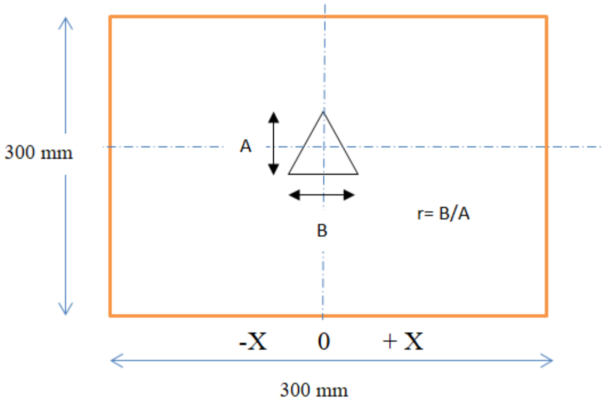


Fig. 9. Schematic geometry sample for free vibration test with cutouts (triangular).

Free vibration experiments were conducted to determine the natural frequency of FG sandwich plates having various geometrical configurations. A specialized vibration test bench was carefully designed to perform modal analysis techniques and examine the properties of rectangular sandwich plates. The test specimens used in the experiments had varying layers, geometrical arrangments (solid, 6, and 11), and hole configurations, including (2, 4, 6, and 8) distributed holes. The experiments were conducted with simply supported boundary conditions, and the specimens were firmly fastened to fixtures. By completing these experiments, researchers hoped to learn more about FGM sandwich plates' vibrational behavior and natural frequencies.

Two accelerometers and a data acquisition device (DAQ NI-6009) comprise the electronic apparatus used in the experiments. One accelerometer is mounted on each face of the sandwich plates, and they are all placed in the center using paraffin wax. Connecting this electronic device to a computer running LabVIEW and SIGVIEW software is required for the measurement setup.

A model of an impulse force hammer (IH-01) is used to measure free vibration. This hammer can instantly apply force to the FG sandwich plate specimen and includes a strain gauge. The panel is excited and given free rein to vibrate thanks to the digital model, which permits the excitation tool to deliver an output per the exerted load [24, 25].

The two displacement sensors display simultaneous oscillations with the same amplitudes as the sandwich specimen vibrates. The sample signal can be acquired thanks to this feature. The testing procedure was repeated on the same model to ensure accuracy. In order to connect the used instruments to a computer, the signals are transferred through the DAQ. This device engages with another software to facilitate info. Transmission.

Microsoft Excel 2019 software is then used to analyze and process the collected data. The Fast Fourier Transform method transforms the signal from the time domain to the frequency domain. SIGVIEW is used for this transformation because it makes analyzing and seeing the signal's frequency components more accessible. The experimental setup for a simply supported FG sandwich plate using the rigged system depicted in Figure 10 is shown.

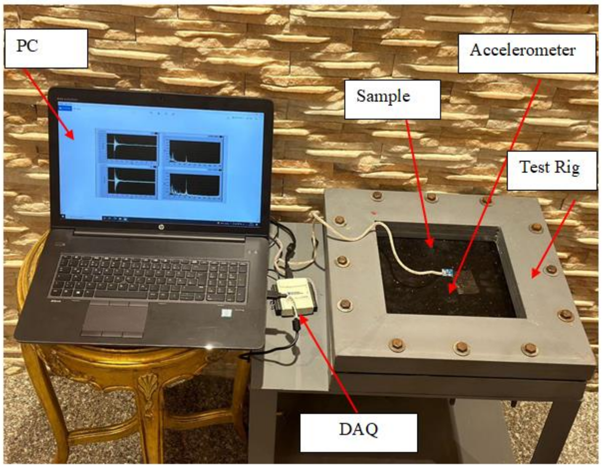


Fig. 10. Free Vibration of sandwich plate test setup.

To sum up, a comprehensive analysis of the vibration signal's frequency domain properties is made possible through acquiring, processing, and examining signals from the deformation transmitters and the digital load part. This analysis involves utilizing various tools, including the NI-60009 DAQ and another program. The same procedure is performed for the FG samples with four types of cutouts. The details of the geometrical properties of a plate with cutouts are explained in Table 4, while Figure 11 shows an FG sample with cutouts. In Figure 12, the view of the LabVIEW software is illustrated as a test sample of the free vibration of the FG sandwich plate.

Fig. 11. FG sample with various cutouts.

Table 5.

The details of cutout samples

Sample No.	Cutout shape	Aspect ratio (r)	Dimensions (mm)	No. of layers
S1	Circular	1	30*30*10	10
S2	Circular	1.5	30*30*10	10
S3	Circular	2	30*30*10	10
S4	Rectangular	1	30*30*10	10
S5	Rectangular	1.5	30*30*10	10
S6	Rectangular	2	30*30*10	10
S7	Triangular	1	30*30*10	10
S8	Triangular	1.5	30*30*10	10
S9	Triangular	1.5	30*30*10	10

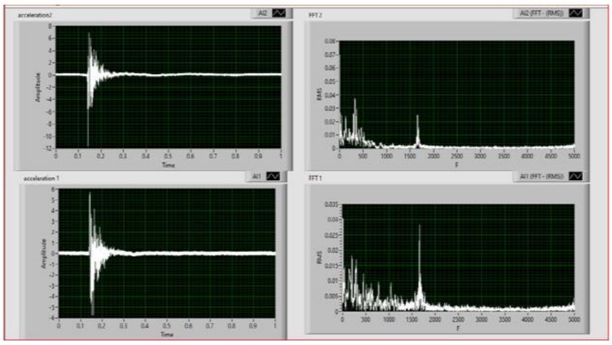


Fig. 12. View of the LabVIEW control panel for a four-hole rectangular plate.

III. Numerical Simulation

A finite element analysis (FEA) with the help of adequate software is used for the numerical analysis of free vibration analysis of sandwich plates [26-29]. Simulation and analysis of the free vibrations of plates are facilitated by ABAQUS, a powerful tool for simulating and studying the dynamic behavior of structures. We can obtain its natural frequencies and associated mode shapes using advanced numerical techniques, helping us understand its dynamic characteristics. The ABAQUS analysis process is divided into three stages: designing, applying boundary conditions, acquiring results, and interpreting them.

The Model Constructing

This work generated a rectangular sandwich plate with different geometrical attributes using SOLIDWORKS 2021 and ABAQUS. This study aims to simulate and analyze the fundamental natural frequencies of six different modes. The dimensions of each modal are 30 cm x 30 cm x 1 cm. The first mode consists of 11 plies made of functionally graded material (FGM), as illustrated in Figure 13a. The second mode also consists of 11 plies, but it includes four holes with a diameter of 1 cm, as shown in Figure 13b. The third mode has exact dimensions but only consists of six layers. The fourth mode is similar to the second mode but with the addition of holes. The fifth mode has the exact dimensions as the first mode but with only four layers. Finally, the sixth mode consists of four layers with four holes.

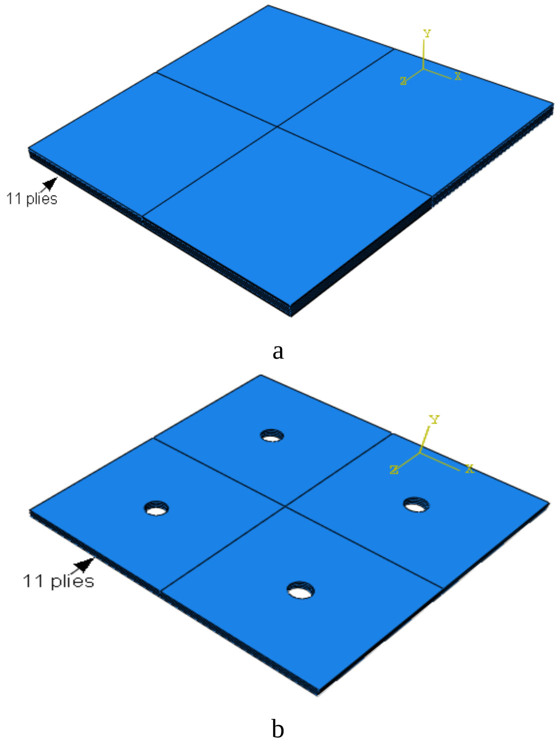


Fig. 13. Rectangular plates model (a) without holes, (b) with four holes.

The Proper Selecting of Element Type

Over 6800 different element forms can be found in the ABAQUS element, a relational database. These elements are divided into groups based on various identification numbers and prefixes. The choice of the model's two- or three-dimensional geometry and the associated degrees of freedom (DOF) largely determines the classification of elements and types. This classification process can find the element type best suited for the desired analysis. The default element in this study is C3D20. It has 20-node quadratic brick and high accuracy and robustness, making it popular for representing structural models. This type of element offers accurate results by providing quadratic interpolation functions within the component. Solid mechanics, structural analysis, and finite element modeling are just a few of the applications it is appropriate for because it can capture complex deformations, linear analysis, and complex nonlinear analyses involving contact, plasticity, and large deformations. This element, which has 20 nodes and is a higher-order substantial element with quadratic spatial properties, is shown in Figure 14. It has three degrees of freedom (DOF) for translations in the typical axes. The C3D20 element allows us to study phenomena such as large deformations, material nonlinearity, contact interactions, and the behavior of solid structures under different loading conditions. With 20 nodes, the element can accurately represent curved boundaries and capture complex structural responses more precisely [ABAQUS help].

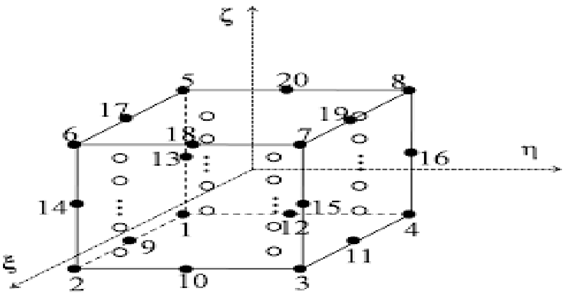


Fig. 14. C3D20 element.

Employing material characteristics

Engineering materials can generally be classified based on various material characteristics depending on their intended use. These properties can be constant, variable, isotropic, orthotropic, or asymmetric. Like element types and fundamental constants, each material property is given a distinct number as a reference for the property set. ABAQUS material properties are frequently obtained through experiments or imported from ABAQUS libraries. For elements kept on the model constructed, isotropic linear and mechanical properties are commonly used [30].

Mesh Generation

In analyzing free vibration in a plate, selecting a suitable mesh plays a vital role in accurately capturing the plate's vibrational behavior. In this case, a mesh of 64,545 nodes and 14,440 quadratic hexahedral elements of type C3D20 was utilized. The mesh generation process aims to discretize the plate geometry into small finite elements, ensuring that the mesh is fine enough to capture the desired vibrational behavior and is computationally efficient.

Choosing an appropriate mesh density is crucial as an overly coarse mesh may result in an inadequate representation of the plate's deformation, while a fine mesh can lead to unnecessarily high computational costs. In this case, the mesh includes quadratic hexahedral elements (C3D20), as shown in Figure 15, providing a higher-order interpolation function for better capturing the plate's vibration modes than linear elements. By employing this suitable mesh configuration, the analysis can effectively capture the complex vibrational behavior of the plate. However, it is essential to note that the specific mesh requirements may vary depending on the plate geometry, material properties, and the desired level of accuracy. Figure 16 shows the status of model constraints in tools software analysis.

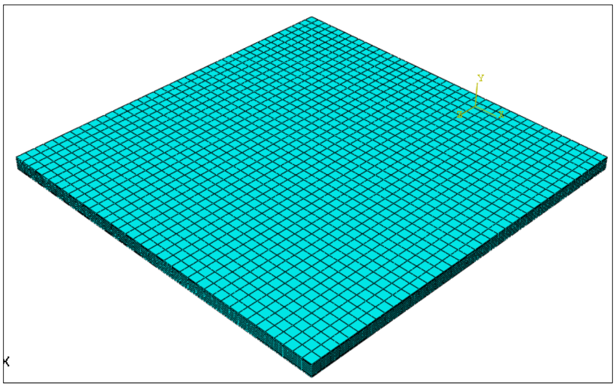


Fig. 15. Simulation of the sandwich porous FGM rectangular plate model.

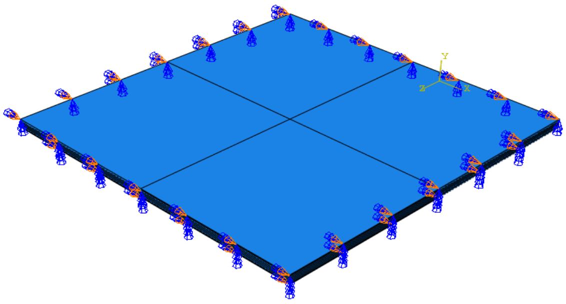


Fig. 16. Status of model constraints in tools software analysis

Simulation Analysis for Free Vibration Modal Investigation

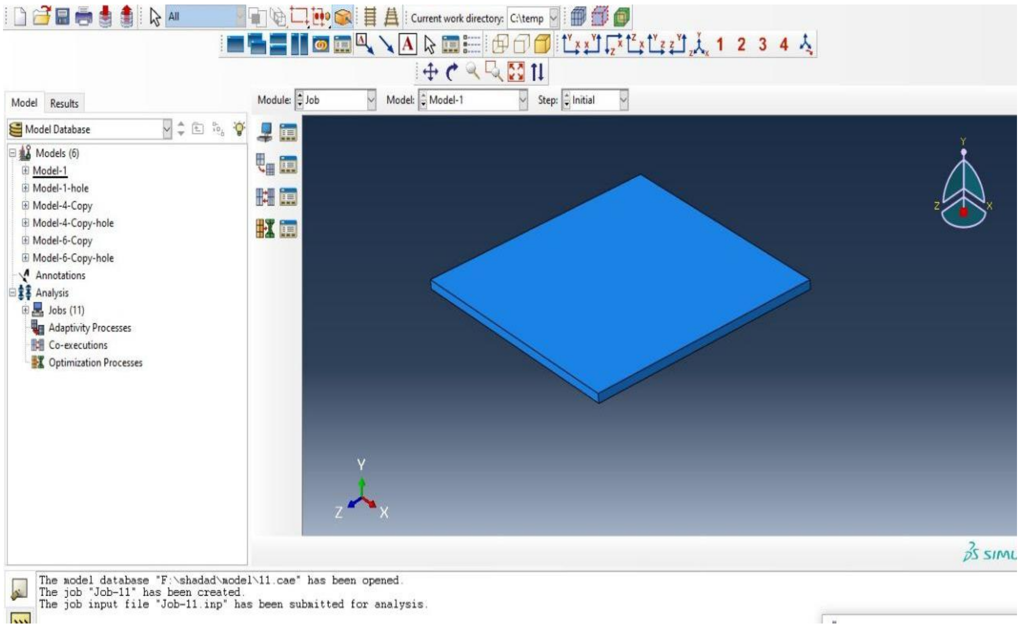
The Lanczos algorithm commences by selecting an initial vector, often a random vector or a mode shape from a previous analysis. This vector is then normalized and utilized as the starting point for the iterative process. At each iteration, the Lanczos algorithm performs a series of matrix-vector multiplications to generate a sequence of vectors that span a Krylov subspace. This subspace captures the essential information to determine the desired eigenvalues and eigenvectors. The Lanczos algorithm efficiently converges towards the desired eigenpairs by orthogonalizing these vectors and constructing a tridiagonal matrix.

The tridiagonal matrix resulting from the Lanczos iterations is diagonalized using well-established eigenvalue solvers to estimate the eigenvalues and eigenvectors. The eigenvalues represent the natural frequencies of the composite plate, while the eigenvectors illustrate the corresponding mode shapes.

Engineers and researchers can evaluate various aspects of the composite plate's dynamic behavior by analyzing the obtained eigenvalues and eigenvectors. The natural frequencies provide valuable information about the resonance frequencies of the structure, while the mode shapes illustrate the vibration patterns. This study aims to comprehensively understand the plate's vibrational characteristics by considering ten natural frequencies and their corresponding mode shapes [31-33]. Figure 17 shows FG plate models with solid plates and circular cutouts.

IV. Results And Discussion

The finite element solutions for the functionally graded material (FGM) and homogeneous element models demonstrate nearly identical results, although they slightly overestimate the analytical reference values. The sandwich plate consists of multilayers with different holes subjected to boundary conditions. The results below exemplify Abaqus analysis for the CCCC sandwich plate solid containing four holes with 11, 6, and 4 layers, respectively. A similar procedure can be implemented for models with various characteristics and boundary conditions. The first case is related to the FGM plate behavior with 11 plies without holes, represented by findings illustrated in Figure 18.



Current work directory: C:\temp

Module: Job
Model: Model-1
Step: Initial

```
The model database "F:\shadad\model\11.cae" has been opened.
The job "Job-11" has been created.
The job input file "Job-11.inp" has been submitted for analysis.
```

Fig. 17. FG Plates models of solid plate.

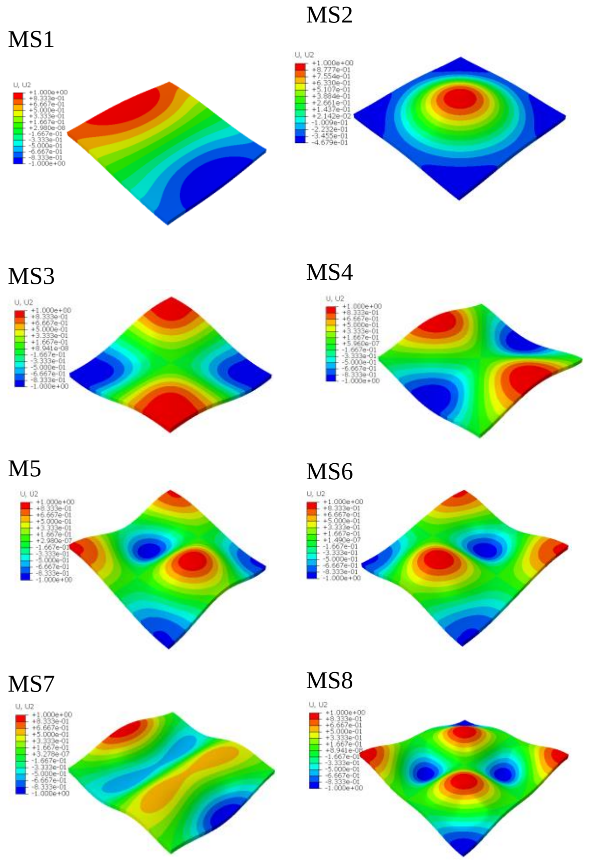


Fig. 18. The first eight mode shapes of the 11-plies model of the CCCC sandwich plate without holes.

The Figure consists of two columns: the mode number and the corresponding fundamental natural frequencies. These frequencies represent the lowest resonant frequencies at which the plate can vibrate in specific mode shapes. In this study, the analysis focused on identifying the plate's fundamental natural frequencies, which indicate its dynamic behavior. Mode number 1 corresponds to the lowest frequency, with a value of 0.00144398. This exceptionally low frequency suggests that this mode represents a global bending or flexural mode shape, where the entire plate undergoes bending or flexing motion. Modes 2 and 3 exhibit the same fundamental natural frequency of 911.359. The similarity in frequency indicates that these modes share similar vibrational behavior.

Further investigation would be necessary to determine the specific mode shapes associated with these frequencies. Similarly, modes 4 and 6 share a fundamental natural frequency of 1194.72, indicating they possess similar mode shapes. The same holds for modes 7 and 8, which have a frequency of 1683.51.

Mode 5 has a fundamental natural frequency of 1071.07, distinct from the other modes analyzed. This frequency suggests a unique mode shape associated with localized vibrations within the plate. Modes 9 and 10 display frequencies of 2142.13 and 2450.08, respectively. These higher frequencies indicate more complex vibrational patterns distinct from the previous modes. It is important to note that several factors, including the plate's material properties, geometry, and boundary conditions, influence the fundamental natural frequencies. These frequencies provide critical information for evaluating the plate's dynamic performance and can help identify potential vibration-related issues. While the Figure offers a concise summary of the results, a more detailed analysis of the mode shapes corresponding to each frequency would be necessary to understand the plate's vibrational behavior comprehensively. Further investigations into the specific mode shapes, their spatial distribution, and the effects of various parameters on the frequencies would contribute to a more thorough analysis and interpretation of the results.

Figure 19 presents the results obtained from the vibration analysis of a plate with a hole; the fundamental natural frequencies reveal distinct mode shapes representing the vibration analysis of an FGM plate composed of 11 layers without holes. Mode 1 has an exceptionally low frequency of 0.00144398, indicating a global bending or flexural mode where the entire plate undergoes bending or flexing motion. Modes 2 and 3 share the same frequency of 877.359, suggesting similar vibrational behavior. Mode 6 also exhibits a frequency of 1370.72, indicating similar mode shapes.

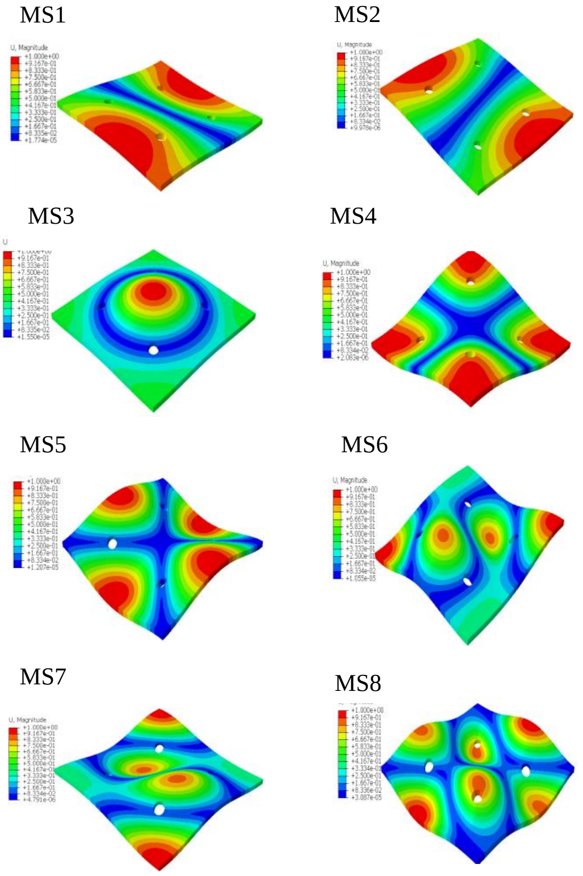


Fig. 19. The first eight mode shapes of natural frequencies of the CCCC sandwich plate contain four holes (11 plies) with four holes.

Similarly, modes 7 and 8 have a frequency of 1693.51, implying similar vibrational patterns. Mode 5 stands out with a frequency of 1071.07, indicating a unique mode shape associated with localized vibrations within the plate. Modes 9 and 10 exhibit higher frequencies of 1642.13 and 2172.08, respectively, showing more complex vibrational patterns distinct from the previous modes. It is important to note that various factors, such as material properties, geometry, and boundary conditions, influence these fundamental natural frequencies.

To evaluate how well the mathematical model predicts the frequency of an FGM sandwich plate made from FGM with various holes and layers, Table 6 presents the (ωn_1) parameter of an SS FGSP. The following formula [34] can be used to calculate the frequency parameter:

$$\psi = \frac{\omega L^2}{h}\sqrt{\frac{D_0}{E_0}} \tag{1}$$

Here, (ω) is the free vibration (ω_{n1}) value that could be estimated ($D_{o.}$= 1000 g/m^3 and E_o=1000 MPa).

Table 6.

Experimental and numerical variation of SSSS rectangular sandwich plate

No. of layers	No. of holes	Num.	Exp.	Discrepancy Numerical and Experimental (%)
4	Solid	0.982	0.9	8.350
	4	0.88	0.825	6.356
	6	0.756	0.718	5.026
	8	0.729	0.68	6.722
6	Solid	0.974	0.890	9.394
	4	0.870	0.800	8.807
	6	0.745	0.707	5.484
	8	0.715	0.687	4.018
11	Solid	0.890	0.844	5.497
	4	0.777	0.720	8.446
	6	0.731	0.695	5.193
	8	0.599	0.563	6.359

According to these results, the (ω_{n1}) of the FGP decreases as the number of holes rises; the logical cause is related to the plate's toughness decreasing as the number of holes increases. One crucial not seen is that the difference in the natural frequency value is 10 % when the plate contains only four holes; this difference becomes 15% by increasing the number of holes to eight. Also, it is found that the maximum discrepancy between numerical and experimental results, no more than 9 %, occurs at a plate with six layers without holes. Furthermore, it is noted that no significant changes occur by increasing the number of layers from 6 to 11, which means the equivalent mechanical properties of the structure exhibit no considerable variation.

Table 7 illustrates the numerical results of frequency parameters obtained by ABAQUS for functionally sandwich plates subjected to four types of boundary conditions (CCCC, CCCS, CSCS, and SSSS) and four aspect ratios (0.5, 0.75, 1, and 2), respectively. A correlation is found between the ψ value and the limitation or restriction in the chosen type. For instance, the frequency parameter in the CCCC model has an aspect ratio of 0.5 at the plate with four holes and with plies number of (n =11) is (0.827), while in CCCS, it was (0.748), and in CSCS (0.719), whereas in SSSC it was (0.545). The results indicate that the frequency parameter increases with the aspect ratio, signifying that the plate behaves more stiffly. However, regarding the number of holes, the same conclusion can be obtained when the plate loses more stiffness with increasing holes (i.e., the possibility of increased porosity inside the plate structure).

Table 7.

Model Numerical results with Four aspect ratios and different boundary conditions of rectangular FG sandwich plates (11 layers)

BCs	No. of holes	Aspect ratio			
		0.5	0.75	1	2
CCCC	Solid	0.852	1.100	1.185	3.325
	4	0.827	0.956	1.045	3.145
	6	0.778	0.942	0.985	3.065
	8	0.759	0.887	0.967	3.017
CCCS	Solid	0.780	1.000	1.077	3.011
	4	0.748	0.869	0.962	2.864
	6	0.710	0.850	0.895	2.792
	8	0.697	0.809	0.876	2.765
CSCS	Solid	0.750	0.966	1.039	2.898
	4	0.719	0.843	0.932	2.749
	6	0.683	0.822	0.864	2.678
	8	0.670	0.781	0.850	2.662
SSSS	Solid	0.568	0.768	0.890	2.559
	4	0.545	0.650	0.777	2.488
	6	0.525	0.645	0.731	2.419
	8	0.508	0.615	0.599	2.398

Table 8 gives the calculations of frequency parameters for various shapes of the cutouts and aspect ratios of FGCP; from the result, one can conclude that by increasing the aspect ratio of the cutouts, the fundamental frequency decreases.

Table 8.

Numerical findings for ψ of square FGSP for multi-cutouts configurations

Cutout type	Aspect ratio	Exp.	Num.	Discrepancy Numerical and Experimental (%)
Circular	0.5	1.764	1.685	4.478
	1	1.728	1.590	7.986
	1.5	1.689	1.586	6.098
	2	1.641	1.492	9.080
	2.5	1.618	1.475	8.838
Rectangular	0.5	1.603	1.459	8.983
	1	1.564	1.430	8.568
	1.5	1.539	1.425	7.407
	2	1.517	1.399	7.779
	2.5	1.492	1.388	6.971
Triangular	0.5	1.516	1.370	9.631
	1	1.490	1.365	8.389
	1.5	1.465	1.355	7.509
	2	1.764	1.685	4.478
	2.5	1.728	1.590	7.986

Conclusion

There are several interpretations can be recorded to sustain the nanocomposite shell, mechanical and natural frequency properties, including:

Nanofillers can enhance composite mechanical properties and free vibration characteristics within a specific concentration of nanoparticles. However, few researchers have studied the free vibration and static behavior of functionally graded nanocomposites reinforced with graphene nanoparticles using polyester resin as a matrix.

The positive effects of the graphene nanoparticles can help improve the composites' static and dynamic properties. However, the functionally graded composite with nanographene fillers under flexural vibration loading conditions has not received enough research attention.

The frequency parameter of the sandwich plate increases as the boundary conditions are tightened, such as when going from CCCS to CCCC. This pattern is consistent with other boundary conditions, such as CSCS.

Graphene nanoparticles improved the natural frequency (25%) compared to solid samples.

Increasing the number of layers from 4 to 11 sustains the value of natural frequency by 17.833 % at a plate with eight holes.

The experimental and FEA results obtained using the ABAQUS software agreed with the flexural and free vibration analysis with a maximum discrepancy of no more than 10%.

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Аналіз вільних вібрацій багатошарових функціонально градуйованих пластин на основі нанокомпозитів поліестер/графен

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Нанокомпозит FGM, виготовлений із багатьох шарів наночастинок графену та матриці на основі поліефіру, створено за допомогою прес-форм із технікою ручного укладання для досягнення точної форми та зменшення дефектів кінцевого продукту. Прогнозовані моделі мають чотири, шість та 11 шарів, а також різні об'ємні частки наночастинок (0,5, 1, 2, 3, 4 та 5%). У цьому дослідженні проведено експериментальні випробування для аналізу характеристик вільної вібрації функціонально градуйованих композитних сендвіч-прямокутних конструкцій для оцінки впливу кількості дірок і локалізації зрізів на основних частотах. Для дірок використовуються три моделі (2, 4 і 6) отворів діаметром 10 мм, а для вирізів використовуються три геометричні конструкції (круглу, прямокутну і трикутну) із різними співвідношеннями сторін (r = 1, 1,5, 2 і 2.5).

Для перевірки експериментального рішення з використанням модального аналізу та аналізу кінцевих елементів (FEA) проведено чисельне дослідження з використанням програмних засобів ABAQUS. Експериментальні висновки та чисельні розрахунки демонструють задовільний рівень збігу із максимальною розбіжністю 9,5 %. Результати показують, що основна частота зменшується зі збільшенням співвідношення сторін вирізів (r = a/b). Існує мінімальна варіація між r =1 і r = 1,5, але помітне зменшення спостерігається при співвідношенні сторін r = 2,5. На таку відмінність впливає, в першу чергу, тип градієнта матеріалу та кількість отворів, зокрема для заданої товщини функціонально градуйованих (FG) пластин.

Keywords: FGM, нанокомпозити, геометричні властивості, частота, ABAQUS.