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Analysis of stress-strain state of the metal plate based on discrete data of displacement values

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An actual problem of stress-strain state (SSS) analysis of a bent metal plate under an uncertain acting load is solved. The practical aspect of the problem is the possibility of reproducing the deformed surface of the plate as a continuous field based on discrete data of displacements at characteristic points of the plate during bending.

A problem in the direct use of spatial interpolation for obtaining the surface of a metal plate, which is realized in the Surfer software product, is formed. It is noted that direct use of existing interpolation methods without considering physical-mechanical properties leads to results distortion. It was noted that spatial interpolation methods have analogy with the finite element method (FEM) in terms of surface display using meshes. This was the basis for comparing the results of surface modelling by FEM and grid-cell interpolation methods. The main difference of the interpolated surface was the absence of spline properties.

A new approach to solving the problem was the additional use of second and fourth-order polynomial spline functions, whose expressions are obtained from the initial and boundary conditions of the deformed plate. With its help, lines of smooth curvature, which acted as frame lines, were obtained and plotted on the grid-cell surface. The introduction of the frame lines on the grid-cell resulted in the combination of the initial discrete data with the displacement values calculated using the expressions of polynomial spline functions. A significant increase in the number of point data allowed the spatial interpolation method to extend the spline properties to the grid-cell interframe space as well. As a result, an adequate deformed surface of a metal plate was reproduced, giving it spline properties - smooth radii of curvature.

Keywords: metal plate, stress-strain state, finite element method, discrete data, spatial interpolation, polynomial spline function, grid-cell.

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Introduction

Metal flat plates have found wide application in engineering constructions, where it is necessary to absorb the load from bulk materials, liquid substances and gaseous mixtures. Plates are the structural material used for manufacturing technical tanks, reservoirs, as well as vertical walls of engineering structures [1].

Metal plates have a rectangular shape of standard dimensions. And continuous walls of structures of arbitrary dimensions are formed from several plates, using non-separable connection between neighbouring plates.

During the operation of constructions, its continuous walls are deformed by loads from the internal environment. Together with the walls, the interconnected plates are also subjected to deformation. The main type of deformation of metal plates is bending.

Thus, metal plates are constantly in a stress-strain state (SSS). Its manifestation is linear displacements of the plate surface points, which are especially noticeable at the corners of a rectangular plate. The values of these displacements are several millimetres and can be recorded by modern measuring devices [2].

These deformation measurements are performed periodically during the operation of the construction to control its technical condition. The surveys produce a set of discrete data is obtained in the form of coordinate changes in the position of the points selected for observation. According to its values, the vertical displacements of these points on the surface of the structure are determined. Based on the collected information, the technical condition of the construction is assessed in general and a conclusion is made on the expediency of its further operation.

It should be noted that the final conclusion is made based on general indicators, such as the inclination of the vertical axis, the change in linear dimensions along individual tiers of the construction. At the same time, the available data in the form of displacements of certain points of the construction could well be used to analyse the SSS of individual plates that make up complete walls.

Availability of the results of metal plate SSS analysis will significantly expand the reliability of conclusions about the technical condition of construction. But the obstacle to such analysis is the difficulty in reproducing the deformed surface based on discrete data of displacement values.

The complex approach to the possible calculation of the metal plate SSS based on discrete data consists not only in in creating an adequate model, but also in implementing methods for its automated processing using a software environment.

I. The problem and research methods

Traditional methods of calculating the SSS of metal plates are modelling the process under the action of a known external load. The modelling process involves the reproduction of all characteristic parameters of the plate, considering the methods of its fastening, the properties of the material itself and the type of external load. The calculation results are implemented in the form of diagrams with surfaces of discrete displacements and internal stresses of the plate [3, 4, 5].

A number of popular software such as SolidWorks, Ansys, Lira [6, 7, 8] are used for modelling the SSS of metal plates. Its algorithms are based on the theory of elasticity and calculations are performed using the finite element method.

These programs belong to solid-state design systems and are used mainly in the development of new construction. The strength of the designed construction is evaluated by the values of maximum stress displacements, the values of which are visualized in diagrams [9].

Despite the versatility and high accuracy of the results obtained, programs of this type are difficult to use at the stage of operation of construction. This is explained by the fact that the process of modelling the SSS of the construction by these software tools requires as input information data on the nature of the external load, with the values of its magnitudes and distribution. Only under these conditions the SSS parameters, namely the values of internal stresses and displacements, can be obtained.

In most cases, especially when using distance methods, survey data record changes in geometric parameters, leaving the external load on the construction undefined at that time. Its result in a set of discrete data, the array of which consists of the points coordinates located on the surface of the deformed body and the values of their displacements due to deformation [2].

It should be noted that at this stage of the research, the external load that led to the appearance of deformation processes, remains without attention. In this case, the SSS parameters of metal structures in bending are estimated by geometric indicators of deformation, namely the radii of surface curvature, comparing it with the allowable [10]. And this requires a corresponding accuracy in reproducing the deformation surface of the curved plate.

So, while the traditional approach to determining the SSS of a plate provides for obtaining its deformed surface under a known external load, in our research, such a surface should be obtained based on discrete displacement data of a group of points distributed on the plate surface.

Problems of such type are successfully solved by geographic information systems (GIS) based on spatial interpolation methods. Realization of such problems is carried out in various GIS software in an automated mode [11].

However, GIS software are oriented on modelling the surfaces of terrestrial areas. As a result, the entire available arsenal of interpolation methods, unfortunately, does not take into account the physical processes of the metal plate deformation. The deformation of metal plates, unlike the earth's surfaces, is subject to completely other laws. The splines properties of the deformed metal plate are its manifestation. Therefore, the direct use of interpolation tools to build a deformed plate surface can lead to incorrect results. And this, in turn, becomes an obstacle to analyse the SSS of the plate using discrete data in the form of displacements.

Because automated methods of plate SSS calculation in such a problem formulation have not been developed yet, it becomes expedient to conduct research on the possibility of using existing software systems that are successfully used in other areas.

GIS software packages will be discussed here primarily. Its choice should correspond to two criteria. First, the ability to reproduce a continuous surface of a distributed feature using discrete data in the grid plane. Second, the ability to perform mathematical operations to compare the obtained results with each step closer to an adequate model.

The Surfer software product was chosen as such an environment, in which spatial interpolation is carried out within a defined grid-cell. Its results are given to the nodes of the grid-cell with subsequent visualization of the three-dimensional surface [12]. There is a direct analogy with the reproduction of a deformed surface by the FEM in the form of a mesh with cells filled with displacement values [6].

Before applying existing algorithms for spatial interpolation within a grid-cell to metal plates, it must first be added with functions for giving spline properties to the deformed surface while observing the radii of curvature [10].

The purpose of the article. To realize the ability to analyse the SSS of metal plates by its surface, which is obtained using the method of spatial interpolation in a grid-cell based on discrete based on discrete displacement data and having spline properties.

Achievement of the set purpose implies the solution of two problems in sequence. In both problems, the object of research is a metal plate with dimensions of

2.4 m × 3.6 m × 0.025 m, which is fixed at both ends by a rigid fastener (Fig. 1).

The first problem will have a test character and is aimed at obtaining a deformed surface, which will act as a reference surface. Therefore, its solution is carried out at a given load by FEM, the results of which are indisputable (Fig. 1a). The second problem will have the opposite case, when the deformations of individual plate points are known, and it will be necessary to reproduce the deformed state of the plate in the form of a surface (Fig. 1b). It should be close to the reference surface.

II. Review of existing approaches to determining the stress-strain state of metal plates

An accurate analysis of the SSS of a thin monolithic plate under the action of a uniformly distributed load acting normal to its surface (Fig. 1a) is reduced to a three-dimensional problem of elasticity theory [13]. At the same time, the use of the well-known Kirchhoff hypotheses allows us to move from a three-dimensional problem to a two-dimensional one, which greatly simplifies its solution [14].

The SSS of flat homogeneous plates in this situation is described by the fourth order differential equation [13]:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q(x,y)}{D} \qquad (1)$$

Differential equation (1) and its corresponding boundary conditions present a mathematical formulation of the problem, whose exact solution within the framework of Kirchhoff theory is well known 14, 15,16]. By integrating the differential equation (1) the analytical dependence of the distribution of strain values $w = w(x, y)$ over the plate surface under the action of the external load $q(x, y)$ is obtained. The character of this distribution is influenced by the constructive parameters of the plate, such as its dimensions (L – length, b – width, t – thickness), stiffness D and methods of its fastening, which are taken into account in the form of boundary conditions.

As different ways of attaching the plate to the base are possible, which lead to changes in the boundary conditions, special approaches to its analytical solution will arise when integrating equation (1) [14, 15, 16]. The situation is further complicated by the fact that the introduction of additional constructive elements on the plate, such as a hole or a rectangular cutout in the middle plates, introduces its own specificity to its analytical solution [4, 8].

In this situation, it is possible to ensure the similarity of the solutions to the set problems by using numerical methods, which are taken as a basis.

One of the most widely used numerical method for solving problems of finding deformed parameters is the FEM. Its advantage is versatility and efficiency in designing new constructions. The main principles of the method, namely the desire to represent the object as continuous medium, are set in the monograph [17].

The algorithm of the discrete approach to calculating the SSS of flat plates involves dividing the whole plate into successive elements in the form of a mesh. The resulting finite elements are characterized by its own properties and interact with neighbouring elements according to deterministic laws. In three-dimensional models the interaction goes through eight nodes, in two-dimensional models - four ones.

Although there is a significant number of supporters of 3D-modelling [18], flat thin plates are usually calculated on 2D-models. A slight loss of accuracy due to the transition to a flat mesh is compensated by the ability to use simpler software tools, including MS Excel. The main advantage of 2D-models should be considered the ability to access two-dimensional arrays with the parameters for further analysis of plate SSS [4].

In conditions of a flat mesh, it becomes possible to develop and use a variety of data analysis algorithms [19]. An example of such algorithms is the problem of finding the minimum potential deformation energy when determining the SSS of structural elements [20].

The following directions can be noted in determining the SSS of plates:

– calculation of plates on different supports (with free

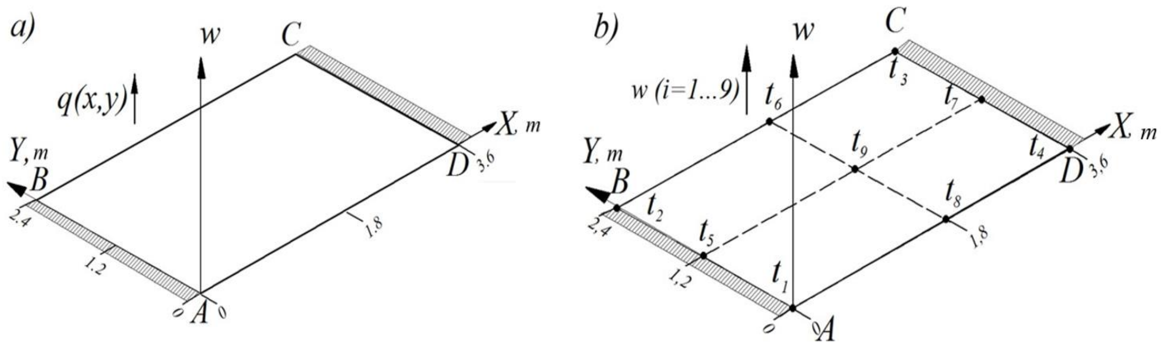

Fig. 1. Calculation schemes for modelling the deformed surface of a metal plate:
a) the first problem – by known external uniformly distributed load $q(x, y)$; b) the second problem – by known displacements of characteristic points w_i, forming a discrete set.

and with rigid mountings) [3];

– influence of constructive modifications of the plate on its SSS in the cases of conducting a square hole [4] and weakening its stiffness by means of holes [8];

– analysis of the SSS of a plate on an elastic base [5].

Transitioning to 2D model, the authors [4] also solved the problem of reducing the number of mesh elements. This led to an increase in the distance between the nodes. Therefore, to ensure the continuity of the surface, it was proposed that the space between the nodes should be approximated by polynomial functions.

The ideas of the combined use of polynomial functions and discrete interpolation methods for obtaining a curved plate surface were formulated in the last century [15]. However, its realization could take place only with the advent of GIS.

Discrete methods of surface construction have been developed in GIS technologies [11]. GIS software (Surfer, etc.) use a grid-cell, the nodes of which are first filled with input data, and then extend to the entire space using spatial interpolation methods. This approach allows to present interpolated data as a model of the territory in three-dimensional space [11, 21]. Spatial interpolation is used to create digital terrain models to research the processes of deformation of the earth's surface during subsidence and landslides [22].

In this context, an interesting article is [23], in which the authors present an algorithm for modelling the deformation of the earth's surface in the form of a polygon by changing the position of its nodal points.

The approaches proposed in GIS may be interesting for visualizing deformed surfaces of metal plates. Its accurate reproduction would allow determining the SSS of the plate based on discrete data of individual point displacements measured during each inspection of technical structures.

However, the problems formulated above require additional research to develop a methodology for determining the SSS of plates based on a grid-cell. During the research it will be necessary to establish the required number of input measurement points, its rational placement on the plate surface, as well as to solve the issue of giving spline properties to deformable plates.

III. Research results

3.1 Comparative analysis of surfaces modelled by FEM and grid-cell

Comparing examples of deformed surfaces of metal plates by FEM [3, 4, 8] with surfaces obtained by GIS [21,22], it is possible to notice certain similarities in the part of visualization of results. Firstly, the given surfaces display deformation in the form of vertical displacements. Secondly, the medium for obtaining surfaces in both cases are meshes based on 2D-models and consist of a set of discrete elements.

Let determine the similarities and differences between FEM and GIS in the way of surface visualization for the purpose of further analysis of deformation processes of metal plates using GIS software. This approach is related to the fact that the problem is solved under conditions of uncertain external load on the based on discrete displacement data.

In comparing the methods, the results of GIS interpolation under grid-cell conditions are of interest because they have similarities with FEM in the representation of surfaces as a matrix equivalent (Fig. 2):

In the two methods of FEM and GIS, the surface of the deformed body is covered with the same type of meshes, its cells are mostly rectangular (Fig. 2 a, b).

In FEM, the cells of the mesh are called finite elements (FE) with nodes labelled as *a, b, c, d* (Fig. 2a). In GIS, a grid-cell generally resembles a spreadsheet processor. Nodes of cells have coordinates of its location along the rows and columns, similar to the matrix elements, and are linked to spatial metric coordinates (Fig. 2b).

The analysis of deformation processes in FEM is considered on the theory that describes the motion of a continuous medium. In this case, the continuum equation is written for the nodes of the finite elements. Therefore, the nodes of the finite element FE (*a, b, c, d*) belong to the plate and are mixed with it during deformation, occupying a new position (*a', b', c', d'*). In this case, the mesh in the FEM is deformed (Fig. 2a).

In a grid-cell conditions, the displacement values are obtained by spatial interpolation methods and become the contents of its nodes. The visualization of interpolated values w_i presents it as the surface $w = w(x, y)$. The analysis of deformed processes is considered in terms of

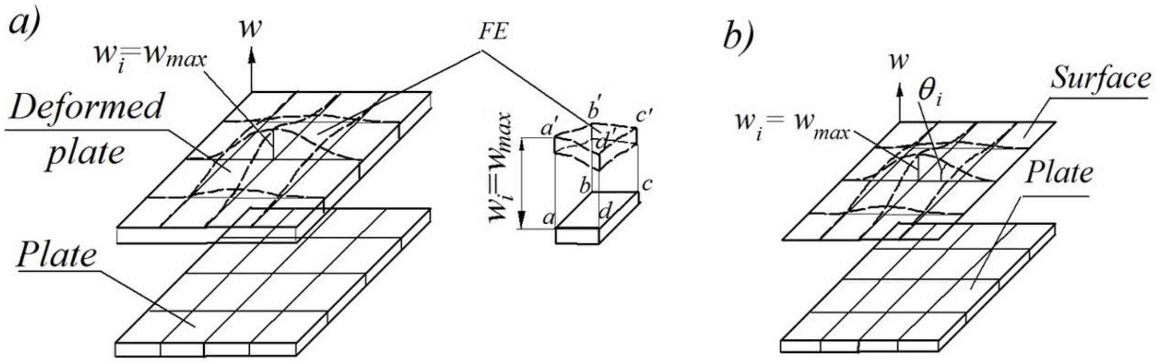

Fig. 2. Deformation surface representation of the plate in the models: deformed mesh in FEM (a) and undeformed grid-cell (b) in GIS.

an undeformed grid (Fig. 2b).

In this way, the FEM is based on the differential equation (1), which for each mesh element is rewritten in algebraic form as the interaction of mesh nodes. At the same time, in GIS, complex processes of deformation of the earth's surface occurring in its interior are attempted to be explained based on changes in the position of characteristic surface points with marked [22, 23].

3.2 FEM modelling of the deformed state of the plate (first task: obtaining a reference surface)

The modelling of the metal plate SSS was performed for two cases: as a solution of the problem of elasticity theory by FEM using the Ansys software [7], and using the interpolation methods of the Surfer program [12]. Although the two problems differed in the parameters of the input data, its results in both cases were reduced to models of deformed plate surfaces.

The purpose of the first problem was to obtain a reference surface of the deformed plate, which was used to evaluate the accuracy of surface reproduction in solving the second problem.

Modelling in Ansys software was started by developing a flat finite element model according to the geometric design scheme (Fig. 1 a).

In the design scheme, the plate is considered as a thin uniform rectangular body under the action of a uniformly distributed load. Its physical and mechanical properties, such as elastic modulus, Poisson's ratio, and specific gravity, were taken from the material management set of the Ansys database.

The main aspect of SSS modelling is the establishment of boundary conditions. The boundary conditions are determined by the way the plate is mounted. Based on the design scheme, the boundary conditions for building the model are formed as follows. Two opposite sides of the plate width have rigid fastening, so when forming the boundary conditions, zero values are assigned to the displacements and angles of rotation of the surface at the plate attachment point.

An important question when modelling SSS is the discretization of space in the form of a finite element mesh. The principles of constructing finite element mesh are determined by the curvature of the deformed surface according to the principle: higher stresses correspond to a smaller radius of curvature, which requires the thickening of the meshes.

In accordance with the research methodology, the finite element mesh must also be the same type as the grid-cell for the interpolation procedure. This will ensure that data can be exchanged between the two programs. Therefore, to solve the first problem, a mesh with finite elements of a square shape having a side of 0.04 m was designed. As a result, the deformable surface of the plate was covered with a mesh size of 61 rows × 91 columns, totalling 5551 finite elements. This grid size will also be used for surface modelling by interpolation.

Following the FEM algorithms, the displacements in the mesh nodes were calculated. After that, it became possible to obtain a 3D-surface deformation in the space limited by the dimensions of the plate.

Considering that the deformed surface obtained by the FEM acted as a reference, it needed to be presented in a format for comparing it with the expected results of the second problem. In this case, the comparison of the modelling results of the two surfaces should be performed in a single software tool.

For this purpose, the software Surfer is best suited with its ability to apply matrix operations between different surfaces that are formed by meshes [12].

Based on the finite element model, an array of discrete values of plate displacements in the form of a 61×91 matrix was formed on the obtained surface. The obtained array was transferred to the Surfer software, where it adopted the format of a coordinated matrix and was visualized as a surface (Fig. 3).

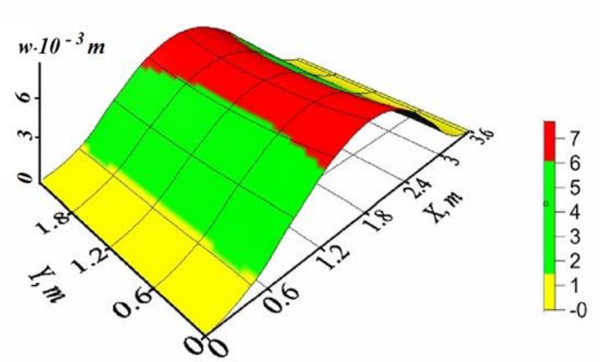

Fig. 3. Graph of the deformed metal plate surface based on the results of solving of the first problem in the Surfer software format.

According to the graph in Fig. 3 the values of displacements of characteristic points are established. Maximum values of $w = 7{,}70 \cdot 10^{-3}$ m were recorded at the points located in the middle of the plate sides ($X = 1.8$ m, $Y = 0$ and $X = 1.8$ m, $Y = 2.4$ m). At the geometric centre of the plate ($X = 1.8$ m at $Y = 1.2$ m), the displacement is slightly smaller and was $w = 6{,}72 \cdot 10^{-3}$ m.

So, the result of solving the first problem was an array of values of plate surface displacements based on a finite element model presented in the Surfer program format as a matrix equivalent. Such representation is suitable for variations of spatial interpolation models at the stage of its improvement.

Its results allow to proceed to the next phase of research related to the solution of the second problem. From the obtained array, the necessary discrete input data are selectively generated to solve the second problem, which addresses the issue of reproducing the surface of a deformed plate by spatial interpolation methods.

3.3 Modelling of the deformed metal plate surface in a grid (second problem – development of an algorithm).

The content of the second problem of determining the SSS of a metal plate is formulated as follows. To research the possibility of obtaining the adequate deformed plate surface from discrete data of displacements of its specific points.

It should be noted that it is not possible to use the differential equation (1) to solve the second problem for several reasons:

1) the distribution function of the external load $q(x, y)$, acting perpendicular to the plate surface is undefined;

2) instead of a continuous deformation field of displacements $w = w(x, y)$, there is only a limited set of discrete data on the displacement values at several specific points w_i.

Therefore, the method of solving the second problem is spatial interpolation, which should reproduce a surface identical to the FEM in a grid-cell using the discrete data.

At the initial stage of the research, the surface was constructed using nine points $t_1 \dots t_9$. The choice of points was dictated by the conditions of its characteristic placement on the plate surface and the ability to measure the value of its displacements under operating conditions at structures. Such points were: four points on the vertices of the rectangular plate ($t_1..t_4$), other four points in the middle of the sides ($t_5..t_8$) and one-point t_9 in the geometric centre of the plate (Fig. 1b).

The first attempt to obtain the deformed plate surface by interpolation methods using only displacement values could not ensure its adequacy. It was found that its main disadvantage was the appearance of kinks on the surface. As a result, the first derivatives at the inflection points had discontinuities, which made it impossible to determine the radius of the surface curvature. It was unrealistic to analyse the SSS using such a model.

At the same time, it illustrated the unresolved problem of obtaining a deformed surface of metal plates by spatial interpolation in a grid-cell. The solution of the first problem by the BEM showed that the deformed plate acquires spline properties with a certain the radius of the surface curvature–during bending. And the radius of curvature is known has a direct analogue of the bending moment and internal stresses in the cross-sections of the plate [10].

The absence of spline properties is explained by the interpolation algorithms used to obtain digital elevation models (DEM). Only the coordinates of selected points are used to build the earth's surface. It is possible to achieve a good index of the surface by standard deviation [24], but not to achieve spline properties.

Returning to equation (1), it can be seen that its solution depends on the initial and boundary conditions. These conditions are formulated for the plate mountings and specify the displacements and the values of the first derivatives, which are the surface inclination angles.

The idea to reproduce the spline properties of the plate in solving the second problem was to additionally apply the boundary conditions written for the angles of rotation of the plate sections in the form of first derivatives (Table 1). However, none of the existing interpolation programs uses the values of first derivatives as input. In fact, first derivatives characterize the flexibility of the surface function better than the displacements.

Further research was conducted in the direction of finding opportunities to give spline properties to existing methods of interpolation on the grid-cell space in the Surfer software. The main accent was made on the use of additional polynomial spline functions, the analytical expressions of which were proposed to be obtained not only based on discrete displacement data, but also taking into account the boundary conditions of plate fixation (Table 1). The boundary conditions for the sides *AB* and *CD* (Fig. 1) will be the values of the first derivatives of the surface function, the geometric content of which is the value of the angles θ_i of its inclination. The angles of inclination at points t_1, t_2, t_3, t_4, t_5, t_7 are determined by the type of plate supports on sides *AB* and *CD* – as rigid fixing. It is known that for a rigid support the tilt angle is zero $\theta_i = 0$ (i = 1..5, 7).

Polynomial spline functions in the form of graphical lines are able to create a preliminary framework on which

Table 1.

Initial and boundary conditions of spline functions formation for grid-cell frame lines

Point Recog-nition	Placement	Displacement values	Initial and boundary conditions
t_1	Side *AB* - rigid fixing	$x=0; \quad y=0$	$w_1=0; \quad \theta_1=\frac{\partial w(x,y)}{\partial x}=0$
t_2		$x=0; \quad y=b$	$w_2=0; \quad \theta_2=\frac{\partial w(x,y)}{\partial x}=0$
t_5		$x=0; \quad y=\frac{b}{2}$	$w_5=0; \quad \theta_5=\frac{\partial w(x,y)}{\partial x}=0$
t_3	Side *CD* - rigid fixing	$x=l; \quad y=b$	$w_3=0; \quad \theta_3=\frac{\partial w(x,y)}{\partial x}=0$
t_4		$x=0; \quad y=b$	$w_4=0; \quad \theta_4=\frac{\partial w(x,y)}{\partial x}=0$
t_7		$x=l; \quad y=\frac{b}{2}$	$w_7=0; \quad \theta_7=\frac{\partial w(x,y)}{\partial x}=0$
t_6	Midpoint of lateral side *BC*	$x=\frac{b}{2}; \quad y=b$	$w_7=7,70\cdot10^{-3}$ м
t_8	Midpoint of lateral side *AD*	$x=\frac{b}{2}; \quad y=0$	$w_8=7,70\cdot10^{-3}$ м
t_9	Geometric centre of the plate	$x=\frac{b}{2}; \quad y=\frac{b}{2}$	$w_7=6,82\cdot10^{-3}$ м

the deformed surface will be stretched by spatial interpolation methods on the shell pattern. For this purpose, it was proposed to divide the grid-cell space covering the plate surface into separate zones, and the dividing lines will be the frame lines. A frame line is a line of smooth curvature that belongs to the plate surface and deforms with it. Each frame line is defined by a higher-order polynomial spline function of higher order.

The described principle was implemented to improve the results of interpolation in the grid-cell conditions of the Surfer software. The spline properties of the metal plate in bending were achieved by the proposed algorithm by pre-treating the input data for successful interpolation process.

The data preparation algorithm includes the following points:

1) placement of initial discrete displacement data, obtaining analytical expressions of polynomial spline functions of the form $w_i = w_i(x, y_i)$ and $w_j = w_j(x_j, y)$ along the direction of the frame lines longitudinal (i = 1..3) and transverse (j = 1);

2) calculation of displacement values by the expressions $w_i = w_i(x, y_i)$ and $w_j = w_j(x_j, y)$ along the entire length of the frame lines with a grid-cell spacing;

3) filling the grid-cell with the obtained values.

As a result, the calculated displacement values are added to the original discrete data and are used together in the interpolation process to form the bending surface.

Fig. 4 shows the placement of the frame lines on the plate surface. In total, four frame lines are used, which can be characterized as longitudinal and transverse. The longitudinal frame lines are located in three planes α, β and γ (i = 1..3), which are perpendicular to the plate surface, and respectively intersect it along the *BC, AD* and longitudinal symmetry axes of the plate. In addition, the plate has a transverse deformation. Its spline properties will be reproduced by the transverse frame line, which is located in the plane δ (j = 1) and coincides with the transverse axis of symmetry of the plate.

Using the boundary and initial conditions from Table 1, the coefficients of the polynomial spline functions were obtained. Its type and coefficient values are shown in Table 2. Depending on the number of boundary and initial conditions, the polynomial functions represented by expressions (2) and (3) are of the fourth or second order.

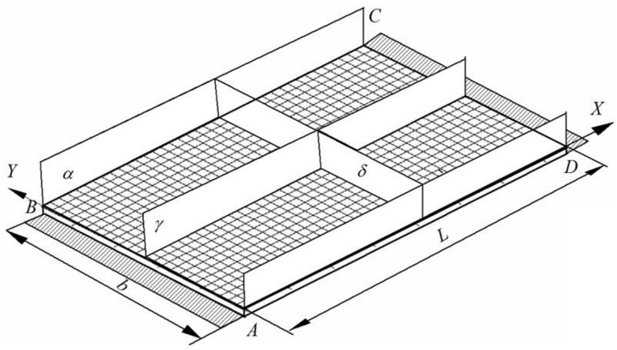

Fig. 4. Placement of frame lines on the grid-cell of the metal plate.

The unknown coefficients of the polynomials (2) and (3) were found using both the displacement values and the derivative values (Table 1). The final analytical polynomial expressions and their coefficients are shown in Table 2.

According to the expressions of polynomials (2) and (3), taking into account the coefficients (Table 2), the values of displacements with discrete grid spacing in the direction of each frame line were calculated. The obtained data filled those the cells of the grid that are located on the frame lines. This procedure combined the initial discrete data with the calculated displacements according to expressions (2) and (3).

The introduction of frame lines divided the plate into separate zones, forming the interframe space. Significant increase in the amount of point data allowed by spatial interpolation methods to extend spline properties to the interframe space of the grid-cell.

Thus, the introduction of frame lines changed the character of interpolation in the interframe space and made it possible to give the bent plate spline properties. As a result, an adequate deformed surface of the metal plate was reproduced, giving it spline properties - smooth radii of curvature. Due the symmetry of the plate and the symmetry of the load, as well as for better visual perception, Fig. 5 shows a ¼ of the surface with a visible profile in cross section.

As can be seen from the graph, the surface contours have acquired smooth properties, the boundary conditions are met in the places of rigid plate attachment, and sharp kinks have been eliminated at extreme displacements.

Table 2.

Polynomial spline function expressions and coefficient values

Plane placement of the frame lines	View of polynomial functions of longitudinal frame lines, (i = 1..3)				
	$w_i(x, y_i) = a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5$ (2)				
	Values of the coefficients				
	a_1	a_2	a_3	a_4	a_5
α, (y = 0)	$7.335 \cdot 10^{-3}$	-0.0528	0.0951	0	0
β, (y = d/2)	$6.497 \cdot 10^{-3}$	-0.047	0.0842	0	0
γ, (y = d)	$7.335 \cdot 10^{-3}$	-0.0528	0.0951	0	0
	View of polynomial functions of the transverse frame line, (j = 1)				
	$w_j(x_j, y) = b_1y^2 + b_2y + b_3$ (3)				
	b_1	b_2	b_3		
δ, (x = L/2)	$6.111 \cdot 10^{-3}$	-0.0146	0.077		

So, account of the boundary conditions of equation (1) with respect to the fastening of the metal plate allowed us to solve the formulated problem and obtain the deformed surface of the plate by discrete data of displacements of specific points of the plate. An important point was the application of polynomial spline function, by means of which the surface acquired spline properties.

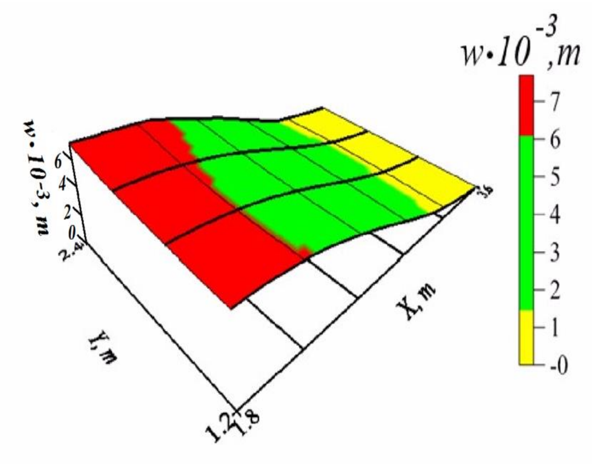

Fig. 5. Deformed surface fragment of a metal plate obtained in grid conditions.

IV. Analysis of research results

The research results are presented in the form of two surfaces obtained by models with fundamentally different approaches (Fig. 2 and Fig. 5). The surface obtained by the first model will be considered as a reference one, since it is the result of the FEM solution. The second model required a series of technical solutions, such as introducing frame lines, obtaining polynomial *spline* functions, and using spatial interpolation methods according to the proposed data preparation algorithm. The efficiency of these steps can be proved by establishing the validity of the obtained surface under grid-cell conditions.

The analysis of the obtained results is aimed at proving the adequacy of the metal plate surface during its bending based on discrete data.

According to the principle of analogy, the comparison of two surfaces was carried out on the Surfer software product.

As the two surfaces were reduced to the same type of grid with a size of 61 × 91, matrix operations were used to check the adequate reproduction, which resulted in the distribution of deviations of the values in terms of both displacements and its relative errors. The similarity of the grid allows not only to compare the displacement values, but also to analyse the spline properties of the surface by the radii of curvature in the longitudinal ρ_x and transverse ρ_y directions.

Fig. 6 compares two graphs of surface deformation in region of maximum displacements with marks of numerical values obtained by the FEM (Fig. 6a) and interpolation methods using frame lines in grid-cell conditions (Fig. 6b).

When comparing the results (Fig. 6), it was found that the relative error in the centre of the plate, where the maximum displacements occurred, was 5–9%, and with the approach to the rigid mount, it increased to 13%, which is explained by the small absolute values of the displacements. The values of the radii of curvature were found to be in agreement with the finite element model. According to the surface data, the radii of curvature at the point of the geometric centre were as follows: in the longitudinal direction ρ_x = 11.9 m, in the transverse direction ρ_y = 81.8 m. And in the longitudinal direction the plate is curved with convexity upwards, and in the transverse direction – downwards.

So, the obtained accuracy indices prove the efficiency of the proposed method for estimating the SSS of a metal plate from discrete strain displacement data.

Conclusions

The actual problem of analysis of the metal plate's SSS based on discrete displacement data under an uncertain external load is solved. An algorithm for constructing the deformed surface of a metal plate in the Surfer software using the methods of spatial interpolation based on a grid-cell is proposed.

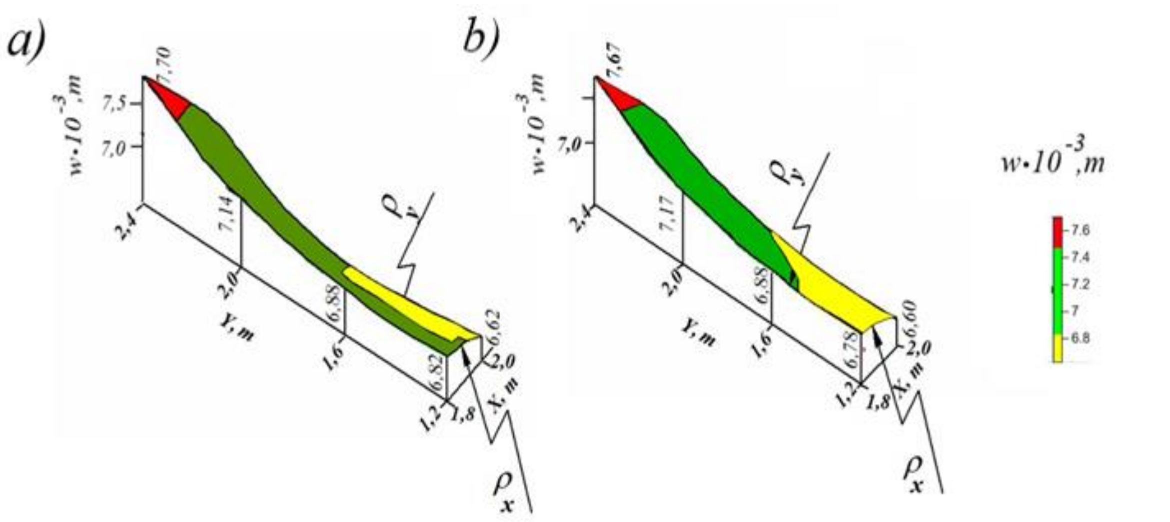

Fig. 6. Graphs of surface deformation in region of maximum displacements: a) FEM model; b) grid-cell condition.

It is shown that the use of polynomial spline functions and the introduction of frame lines on the grid-cell led to a change in the character of interpolation and made it possible to give the bent plate spline properties. By the noted analogy with the FEM, the adequacy of the surface reproduction was confirmed and the values of the radii of curvature were given: in the longitudinal direction the plate is bent with the convexity upward, and in the transverse direction – downward.

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Є.Й. Ріпецький, Р.Й. Ріпецький, О.І. Непеляк

Аналіз напружено-деформованого стану металевої пластини на основі дискретних даних величин переміщень

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Розв’язана актуальна задача аналізу напружено-деформованого стану (НДС) зігнутої металевої пластини в умовах невизначеного діючого навантаження. Практичний аспект задачі полягає в можливості відтворення деформованої поверхні пластини – як суцільного поля за дискретними даними величин переміщень її характерних точок при згині.

Сформульовано проблематику у безпосередньому використанні просторової інтерполяції для отримання поверхні металевої пластини, реалізацію якої здійснено у програмному продукті Surfer. Відмічено, що методи просторової інтерполяції мають аналогію з методом кінцевих елементів (МКЕ) в частині відображення поверхонь з використанням сіток. Це стало підставою для порівнянь результатів моделювання поверхонь за МКЕ з методами інтерполяції в умовах grid-сітки. Основна відмінність інтерпольованої поверхні полягала у відсутності у неї сплайнових властивостей.

Новий підхід у розв'язку поставленої проблеми полягав у додатковому використанні поліноміальних сплайн-функцій другого та четвертого порядків, вирази яких отримують за початковими та граничними умовами деформованої пластини. За їх допомогою було отримано і нанесено на поверхню grid-сітки лінії плавної кривизни, які виступали у ролі каркасних ліній. З введенням каркасних ліній на grid-сітці відбулося поєднання початкових дискретних даних зі значеннями переміщень, які були розраховані за виразами поліноміальних сплайн-функцій. Значне збільшення кількості точкових даних дозволило методу просторової інтерполяції поширити сплайнові властивості і на міжкаркаснй простір grid-сітки. В результаті було відтворено адекватну деформовану поверхню металевої пластини з наданням їй сплайнових властивостей – плавних радіусів кривизни.

Ключові слова: металева пластина, напружено-деформований стан, метод кінцевих елементів, дискретні данні, просторова інтерполяція, поліноміальна сплайн-функція, grid-сітка.